

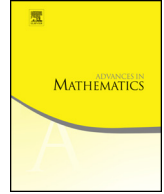


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



# Homological stability for topological chiral homology of completions



Alexander Kupers<sup>a,\*</sup>, Jeremy Miller<sup>b</sup>

<sup>a</sup> Department of Mathematics, Stanford University, 450 Serra Mall, 94305, Stanford, CA, USA

<sup>b</sup> Department of Mathematics, Purdue University, 150 N. University Street, 47907-2067, West Lafayette, IN, USA

## ARTICLE INFO

### Article history:

Received 20 November 2013

Received in revised form 1 February 2016

Accepted 1 February 2016

Communicated by the Managing Editors of AIM

### Keywords:

Homological stability

Topological chiral homology

$E_n$  algebras

Configuration spaces

Symmetric powers

## ABSTRACT

By proving that several new complexes of embedded disks are highly connected, we obtain several new homological stability results. Our main result is homological stability for topological chiral homology on an open manifold with coefficients in certain partial framed  $E_n$ -algebras. Using this, we prove a special case of a conjecture of Vakil and Wood on homological stability for complements of closures of particular strata in the symmetric powers of an open manifold and we prove that the bounded symmetric powers of closed manifolds satisfy homological stability rationally.

© 2016 Elsevier Inc. All rights reserved.

## Contents

1. Introduction	756
2. Definition of topological chiral homology of partial algebras	763
3. Highly connected bounded charge complexes	778
4. Homological stability for completions	802

\* Corresponding author.

E-mail addresses: [kupers@stanford.edu](mailto:kupers@stanford.edu) (A. Kupers), [jeremykmiller@purdue.edu](mailto:jeremykmiller@purdue.edu) (J. Miller).

<sup>1</sup> Alexander Kupers is supported by a William R. Hewlett Stanford Graduate Fellowship, Department of Mathematics, Stanford University, and was partially supported by NSF grant DMS-1105058.

5. The scanning map and closed manifolds . . . . .	812
6. Rational homological stability for bounded symmetric powers of closed manifolds . . . . .	816
Acknowledgments . . . . .	825
References . . . . .	825

---

**1. Introduction**

In this paper we prove a generalization of homological stability for configuration spaces. Let  $M$  be a manifold and let  $C_k(M)$  denote the configuration space of  $k$  distinct unordered particles in  $M$ . If  $M$  is open, then there is a map  $t : C_k(M) \rightarrow C_{k+1}(M)$  adding a particle near infinity; homological stability for configuration spaces says that this map is an isomorphism in homology in a range tending to infinity with  $k$ .

Our generalization involves certain configuration spaces with summable labels. For example, if  $A$  is a commutative monoid, then we consider spaces of particles in  $M$  with labels in  $A$  topologized such that if the particles collide we add their labels. However, such a construction makes sense in a more general setting; the labels only need to have the structure of a so-called *framed  $E_n$ -algebra* [33,15]. The analogous construction of the labeled configuration space is then called *topological chiral homology* [32] and is denoted  $\int_M A$ . It is also known as *factorization homology* [2] or *configuration spaces of particles with summable labels* [45].

We will consider framed  $E_n$ -algebras  $A$  with  $\pi_0(A) = \mathbb{N}_0$  that are “generated by finitely many components,” a notion made precise using *completions* of framed  $E_n$ -algebras. If  $\pi_0(A) = \mathbb{N}_0$  the connected components of  $\int_M A$  are in bijection with  $\mathbb{N}_0$  for connected  $M$  and we denote the  $k$ th component by  $\int_M^k A$ . When  $M$  is open, there is again a stabilization map

$$t : \int_M^k A \rightarrow \int_M^{k+1} A.$$

The main result of this paper is that for  $A$  generated by finitely many components, this map induces an isomorphism in homology in a range tending to infinity with  $k$ .

*1.1. Topological chiral homology*

We start by introducing the spaces we are interested in, which are defined using topological chiral homology.

Topological chiral homology is a homology theory for  $n$ -dimensional manifolds. When discussing topological chiral homology, one needs to fix a background symmetric monoidal  $(\infty, 1)$ -category like chain complexes or spectra. In this paper, this category will always be taken to be **Top**, the category of topological spaces with Cartesian product.

Download English Version:

<https://daneshyari.com/en/article/6425293>

Download Persian Version:

<https://daneshyari.com/article/6425293>

[Daneshyari.com](https://daneshyari.com)