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The representation type of Jacobian algebras

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A B S T R A C T

We show that the representation type of the Jacobian algebra $\mathcal{P}(Q, S)$ associated to a 2-acyclic quiver Q with non-degenerate potential S is invariant under QP-mutations. We prove that, apart from very few exceptions, $\mathcal{P}(Q, S)$ is of tame representation type if and only if Q is of finite mutation type. We also show that most quivers Q of finite mutation type admit only one non-degenerate potential up to weak right equivalence. In this case, the isomorphism class of $\mathcal{P}(Q, S)$ depends only on Q and not on S.

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1. Introduction

1.1. Cluster algebras and Jacobian algebras

Cluster algebras were invented by Fomin and Zelevinsky [22] in an attempt to obtain a combinatorial approach to the dual of Lusztig's canonical basis of quantum groups. Another motivation for cluster algebras was the concept of total positivity, which dates back about 80 years, and was generalized and connected to Lie theory by Lusztig in 1994, see [40,41]. By now numerous connections between cluster algebras and other branches of mathematics have been discovered, e.g. representation theory of quivers and algebras and Donaldson–Thomas invariants of 3-Calabi–Yau categories.

By definition, the *cluster algebra* $\mathcal{A}_B$ associated to a skew-symmetrizable integer matrix B is the subalgebra of a field of rational functions generated by an inductively constructed set of so-called cluster variables. (Starting with B and a set of initial cluster variables the other cluster variables are obtained via iterated *seed mutations*.) In this paper, we assume that the matrix B is skew-symmetric (for arbitrary skew-symmetrizable matrices B the theory of cluster algebras is much less developed). The set of skew-symmetric integer matrices corresponds bijectively to the set of 2-acyclic quivers. We denote $\mathcal{A}_Q := \mathcal{A}_B$ if the quiver Q corresponds to the skew-symmetric matrix B. The set of cluster algebras $\mathcal{A}_Q$ can be divided naturally into two classes. Namely, the quiver Q is either of finite or of infinite mutation type.

One of the main links between the theory of cluster algebras and the representation theory of algebras is given by the work of Derksen, Weyman and Zelevinsky [15,16] on quivers with potentials (Q, S) and the representations of the associated Jacobian algebras $\mathcal{P}(Q, S)$. Here S is a non-degenerate potential on a 2-acyclic quiver Q, and $\mathcal{P}(Q, S)$ is by definition the completed path algebra of Q modulo the closure of the ideal generated by the cyclic derivatives of S. Our first main result (Theorem 1.2) shows that the mutation type of Q is closely related to the representation type of $\mathcal{P}(Q, S)$.

Let $\Gamma := \Gamma(Q, S)$ be the Ginzburg dg-algebra associated to (Q, S), see [30] and also the survey article [32]. The derived category $\mathcal{D}_{\mathrm{f.d.}}(\Gamma)$ of dg-modules over Γ with finite-dimensional total homology is a 3-Calabi–Yau category. It has a natural t-structure whose