

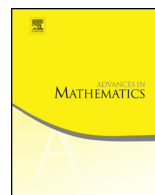


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Congruence properties for a certain kind of partition functions

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ABSTRACT

In light of the modular equations of fifth and seventh order, we derive some congruence properties for a certain kind of partition functions $a(n)$ which satisfy $\sum_{n=0}^{\infty} a(n)q^n \equiv (q; q)_{\infty}^k \pmod{m}$, where k is a positive integer with $1 \leq k \leq 24$ and $m = 2, 3$. In view of these properties, we obtain many infinite families of congruences for $c\phi_k(n)$, the number of generalized Frobenius partitions of n with k colors, and $\overline{c\phi}_k(n)$, the number of generalized Frobenius partitions of n with k colors whose order is k under cyclic permutation of the k colors. Meanwhile, we also apply the main theorems to some other kinds of partition functions.

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1. Introduction

The object of this paper is to discuss congruence properties for a certain kind of partition functions $a(n)$ which satisfy

$$\sum_{n=0}^{\infty} a(n)q^n \equiv f^k(-q) \pmod{m}, \tag{1.1}$$

where k and m are positive integers. Here we use the following notations

$$(a; q)_{\infty} := \prod_{k=0}^{\infty} (1 - aq^k) \quad \text{and} \quad f(-q) := (q; q)_{\infty}.$$

By means of the modular equations of fifth and seventh order, we find a general method to obtain some infinite families of congruences for $a(n)$. In this paper, we mainly focus on the cases for $1 \leq k \leq 24$ and $m = 2, 3$. For large k and other values of m , we can also use the method to search for congruences for $a(n)$. The main results are stated as follows.

Theorem 1.1. *If $\sum_{n=0}^{\infty} a(n)q^n \equiv f^k(-q) \pmod{2}$ with $1 \leq k \leq 24$, then for $\alpha \geq 1$, $n \geq 0$, and $i = 1, 2, 3, 4$, we have*

$$a\left(5^{t\alpha}n + \frac{(24i + 5k) \cdot 5^{t\alpha-1} - k}{24}\right) \equiv 0 \pmod{2},$$

where

$$t = \begin{cases} 2, & k = 1, 2, 3, 4, 6, 7, 8, 9, 12, 13, 14, 16, 18, 19, 24, \\ 4, & \text{otherwise.} \end{cases}$$

Theorem 1.2. *If $\sum_{n=0}^{\infty} a(n)q^n \equiv f^k(-q) \pmod{3}$ with $1 \leq k \leq 24$, then for $\alpha \geq 1$, $n \geq 0$, and $i = 1, 2, 3, 4$, we have*

$$a\left(5^{t\alpha}n + \frac{(24i + 5k) \cdot 5^{t\alpha-1} - k}{24}\right) \equiv 0 \pmod{3},$$

where

$$t = \begin{cases} 2, & \text{if } k = 1, 2, 3, 4, 6, 8, 9, 12, 14, 18, 24, \\ 4, & \text{if } k = 5, 7, 11, 13, 15, 17, 19, 21, 23, \\ 6, & \text{otherwise.} \end{cases}$$

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