Advances in Mathematics 285 (2015) 72–100

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Colocalizations of noncommutative spectra and bootstrap categories ☆

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ARTICLE INFO

Article history:
Received 2 January 2015
Received in revised form 25 July 2015
Accepted 30 July 2015
Available online 24 August 2015
Communicated by Henning Krause

MSC:
19Dxx
46L85
55Nxx
55P42
46L35

Keywords:
Noncommutative spectra
Stable ∞-categories
Triangulated categories
Bootstrap categories
(Co)localizations
C^*-algebras
Bivariant K-theory

ABSTRACT

We construct a compactly generated and closed symmetric monoidal stable ∞-category $\mathtt{NSp}'$ and show that $\mathtt{hNSp}'^{\mathrm{op}}$ contains the suspension stable homotopy category of separable C^*-algebras $\mathtt{SHo}^{C^*}$ constructed by Cuntz–Meyer–Rosenberg as a fully faithful triangulated subcategory. Then we construct two colocalizations of $\mathtt{NSp}'$, namely, $\mathtt{NSp}'[\mathbb{K}^{-1}]$ and $\mathtt{NSp}'[\mathbb{Z}^{-1}]$, both of which are shown to be compactly generated and closed symmetric monoidal. We prove that Kasparov KK-category of separable C^*-algebras sits inside the homotopy category of $\mathtt{KK}_\infty := \mathtt{NSp}'[\mathbb{K}^{-1}]^{\mathrm{op}}$ as a fully faithful triangulated subcategory. Hence $\mathtt{KK}_\infty$ should be viewed as the stable ∞-categorical incarnation of Kasparov KK-category for arbitrary pointed noncommutative spaces (including nonseparable C^*-algebras). As an application we find that the bootstrap category in $\mathtt{hNSp}'[\mathbb{K}^{-1}]$ admits a completely algebraic description. We also construct a K-theoretic bootstrap category in $\mathtt{hKK}_\infty$ that extends the construction of the UCT class by Rosenberg–Schochet. Motivated by the algebraization problem we finally analyze a couple of equivalence relations on separable C^*-algebras that are introduced via the bootstrap categories in various colocalizations of $\mathtt{NSp}'$.

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☆ This research was supported by the Deutsche Forschungsgemeinschaft (SFB 878 and SFB 1085), ERC through AdG 267079, and the Alexander von Humboldt Foundation (Humboldt Professorship of Michael Weiss).

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http://dx.doi.org/10.1016/j.aim.2015.07.029
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Contents

0. Introduction

In [25] we constructed a compactly generated stable ∞-category of noncommutative spectra $\mathtt{NSp}$ primarily with the intention of proving that the noncommutative stable homotopy category is topological according to the definition in [39]. The stable ∞-category $\mathtt{NSp}$ affords an ideal framework for the stable homotopy theory of noncommutative spaces. In [27] we demonstrated that $\mathtt{NSp}$ is closed symmetric monoidal, which enabled us to produce smashing colocalizations of $\mathtt{NSp}$ that generalize bivariant (connective) E-theory category and some variants thereof. One aim of our project is to construct similar interesting stable ∞-subcategories of noncommutative spectra (possibly by colocalizations) and ideally give purely algebraic descriptions of their homotopy categories. This is the *algebraization problem* that pertains to computational aspects. Concurrently this project also settles the long-standing problem of constructing generalized (co)homology theories on the category of C^*-algebras. In fact, thanks to Brown representability in this setup (see Theorem 2.23 of [25] and Remark 1.7), noncommutative spectra parametrize all generalized (co)homology theories. The crucial property is the carefully designed compact generation of noncommutative spectra.

Amongst the bivariant homology theories available in the literature KK-theory plays a distinguished role as it has proved to be remarkably effective in tackling various problems in topology and geometry (see, for instance, [19,35,5,15,44]). The assumption of separability is inherent in Kasparov's original definition of KK-theory [18,17]. However, for certain applications to index theory and problems in non-metrizable topology this assumption is too restrictive. Moreover, the construction of the Kasparov (composition) product is a very delicate issue in this setup. Extending the Cuntz picture of KK-theory it is possible to construct a KK-theory $\mathtt{kk}^{\mathtt{C}^*}$ for nonseparable C^*-algebras (see, for instance, [8]), where the composition product can be established quite easily. In Remark 8.29 of [8] the authors state

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