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Weak expansion properties and large deviation principles for expanding Thurston maps

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ABSTRACT

In this paper, we prove that an expanding Thurston map $f\colon S^2 \to S^2$ is asymptotically h-expansive if and only if it has no periodic critical points, and that no expanding Thurston map is h-expansive. As a consequence, for each expanding Thurston map without periodic critical points and each real-valued continuous potential on S^2, there exists at least one equilibrium state. For such maps, we also establish large deviation principles for iterated preimages and periodic points. It follows that iterated preimages and periodic points are equidistributed with respect to the unique equilibrium state for an expanding Thurston map without periodic critical points and a potential that is Hölder continuous with respect to a visual metric on S^2.

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1. Introduction

The theory of discrete-time dynamical systems studies qualitative and quantitative properties of orbits of points in a space under iterations of a given map. Various conditions can be imposed upon the map to simplify the orbit structures, which in turn lead to results about the dynamical system under consideration. One such well-known condition is expansiveness. Roughly speaking, a map is expansive if no two distinct orbits stay close forever. Expansiveness plays an important role in the exploitation of hyperbolicity in smooth dynamical systems, and in complex dynamics in particular (see for example, [27] and [39]).

In the context of continuous maps on compact metric spaces, there are two weaker notions of expansion, called *h-expansiveness* and *asymptotic h-expansiveness*, introduced by R. Bowen [5] and M. Misiurewicz [30], respectively. Forward-expansiveness implies h-expansiveness, which in turn implies asymptotic h-expansiveness [31]. Both of these weak notions of expansion play important roles in the study of smooth dynamical systems (see [7,14,17,18,25]). Moreover, any smooth map on a compact Riemannian manifold is asymptotically h-expansive [8]. Recently, N.-P. Chung and G. Zhang extended these concepts to the context of a continuous action of a countable discrete sofic group on a compact metric space [10].

The dynamical systems that we study in this paper are induced by *expanding Thurston maps*, which are a priori not differentiable. Thurston maps are branched covering maps on the sphere S^2 that generalize rational maps with finitely many postcritical points on the Riemann sphere. More precisely, a (non-homeomorphic) branched covering map $f\colon S^2 \to S^2$ is a Thurston map if it has finitely many critical points each of which is preperiodic. These maps arose in W.P. Thurston's characterization of postcritically-finite rational maps (see [15]). See Section 3 for a more detailed introduction to Thurston maps.

Inspired by the analogy to Cannon's conjecture in geometric group theory (see for example, [3, Section 5 and Section 6]), M. Bonk and D. Meyer investigated extensively

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