Advances in Mathematics 285 (2015) 658–708

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Operadic multiplications in equivariant spectra, norms, and transfers

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ARTICLE INFO

Article history:
Received 1 October 2013
Received in revised form 23 June 2015
Accepted 21 July 2015
Available online 31 August 2015
Communicated by the Managing Editors of AIM

Keywords:
Equivariant stable homotopy theory
Operads

ABSTRACT

We study homotopy-coherent commutative multiplicative structures on equivariant spaces and spectra. We define N_∞ operads, equivariant generalizations of E_∞ operads. Algebras in equivariant spectra over an N_∞ operad model homotopically commutative equivariant ring spectra that only admit certain collections of Hill–Hopkins–Ravenel norms, determined by the operad. Analogously, algebras in equivariant spaces over an N_∞ operad provide explicit constructions of certain transfers. This characterization yields a conceptual explanation of the structure of equivariant infinite loop spaces.

To explain the relationship between norms, transfers, and N_∞ operads, we discuss the general features of these operads, linking their properties to families of finite sets with group actions and analyzing their behavior under norms and geometric fixed points. A surprising consequence of our study is that in stark contract to the classical setting, equivariantly the little disks and linear isometries operads for a general incomplete universe U need not determine the same algebras. Our work is motivated by the need to provide a framework to describe the flavors of commutativity seen in recent work of

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[1] A.J. Blumberg was supported in part by NSF grants DMS-0906105 and DMS-1151577.
[2] M.A. Hill was supported in part by NSF DMS-1207774, the Sloan Foundation, and by DARPA through the Air Force Office of Scientific Research (AFOSR) grant number HR0011-10-1-0054.

http://dx.doi.org/10.1016/j.aim.2015.07.013
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the second author and Hopkins on localization of equivariant commutative ring spectra.

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Contents

1. Introduction

One of the most important ideas in modern stable homotopy theory is the notion of a structured ring spectrum, an enhancement of the representing object for a multiplicative cohomology theory. A structured ring spectrum is a spectrum equipped with a homotopy-coherent multiplication; classically the coherence data is packaged up in an operad. When the multiplication is coherently commutative (as in the familiar examples of $H\mathbb{Z}$, ku, and MU), the classical operadic description of the multiplication involves an E_∞ operad.

May originally observed that all E_∞ operads are equivalent up to a zig-zag of maps of operads [17] and showed that equivalent E_∞ operads have equivalent homotopical categories of algebras. As an elaboration of this basic insight it is now well-understood that all possible notions of commutative ring spectrum agree. For instance, in the symmetric monoidal categories of EKMM S-modules [5] and of diagram spectra [16] (i.e., symmetric spectra and orthogonal spectra), the associated categories of commutative monoids are homotopically equivalent to the classical category of E_∞-ring spectra [18,14]. Moreover, the homotopy theories of the categories of commutative monoids are equivalent to the homotopy theories of the category of algebras over any reasonable E_∞ operad [5, §II.4].

Our focus in this paper is on equivariant generalizations of E_∞ ring spectra. At first blush, it might seem that we can give an analogous account of the situation. After all, for any compact Lie group G and universe U of finite dimensional G-representations, there is the classical notion of an equivariant E_∞ ring spectrum structured by the equivariant linear isometries operad on U [14]. For each U, there are equivariant analogues of the modern categories of spectra (i.e., equivariant orthogonal spectra and equivariant S-modules) that are symmetric monoidal categories [15,10]. Moreover, once again

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