

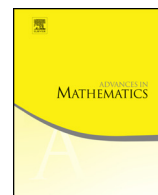


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# Operadic multiplications in equivariant spectra, norms, and transfers

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## ABSTRACT

We study homotopy-coherent commutative multiplicative structures on equivariant spaces and spectra. We define  $N_\infty$  operads, equivariant generalizations of  $E_\infty$  operads. Algebras in equivariant spectra over an  $N_\infty$  operad model homotopically commutative equivariant ring spectra that only admit certain collections of Hill–Hopkins–Ravenel norms, determined by the operad. Analogously, algebras in equivariant spaces over an  $N_\infty$  operad provide explicit constructions of certain transfers. This characterization yields a conceptual explanation of the structure of equivariant infinite loop spaces.

To explain the relationship between norms, transfers, and  $N_\infty$  operads, we discuss the general features of these operads, linking their properties to families of finite sets with group actions and analyzing their behavior under norms and geometric fixed points. A surprising consequence of our study is that in stark contrast to the classical setting, equivariantly the little disks and linear isometries operads for a general incomplete universe  $U$  need not determine the same algebras. Our work is motivated by the need to provide a framework to describe the flavors of commutativity seen in recent work of

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the second author and Hopkins on localization of equivariant commutative ring spectra.

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## 1. Introduction

One of the most important ideas in modern stable homotopy theory is the notion of a structured ring spectrum, an enhancement of the representing object for a multiplicative cohomology theory. A structured ring spectrum is a spectrum equipped with a homotopy-coherent multiplication; classically the coherence data is packaged up in an operad. When the multiplication is coherently commutative (as in the familiar examples of  $H\mathbb{Z}$ ,  $ku$ , and  $MU$ ), the classical operadic description of the multiplication involves an  $E_\infty$  operad.

May originally observed that all  $E_\infty$  operads are equivalent up to a zig-zag of maps of operads [17] and showed that equivalent  $E_\infty$  operads have equivalent homotopical categories of algebras. As an elaboration of this basic insight it is now well-understood that all possible notions of commutative ring spectrum agree. For instance, in the symmetric monoidal categories of EKMM  $S$ -modules [5] and of diagram spectra [16] (i.e., symmetric spectra and orthogonal spectra), the associated categories of commutative monoids are homotopically equivalent to the classical category of  $E_\infty$ -ring spectra [18,14]. Moreover, the homotopy theories of the categories of commutative monoids are equivalent to the homotopy theories of the category of algebras over any reasonable  $E_\infty$  operad [5, §II.4].

Our focus in this paper is on equivariant generalizations of  $E_\infty$  ring spectra. At first blush, it might seem that we can give an analogous account of the situation. After all, for any compact Lie group  $G$  and universe  $U$  of finite dimensional  $G$ -representations, there is the classical notion of an equivariant  $E_\infty$  ring spectrum structured by the equivariant linear isometries operad on  $U$  [14]. For each  $U$ , there are equivariant analogues of the modern categories of spectra (i.e., equivariant orthogonal spectra and equivariant  $S$ -modules) that are symmetric monoidal categories [15,10]. Moreover, once again

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