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Convergence rate, location and ∂_z^2 condition for fully bubbling solutions to $SU(n+1)$ Toda systems

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A B S T R A C T

It is well known that the study of $SU(n+1)$ Toda systems is important not only to Chern–Simons models in Physics, but also to the understanding of holomorphic curves, harmonic sequences or harmonic maps from Riemann surfaces to $\mathbb{CP}^n$. One major goal in the study of $SU(n+1)$ Toda system on Riemann surfaces is to completely understand the asymptotic behavior of fully bubbling solutions. In this article we use a unified approach to study fully bubbling solutions to general $SU(n+1)$ Toda systems and we prove three major sharp estimates important for constructing bubbling solutions: the closeness of blowup solutions to entire solutions, the location of blowup points and a ∂_z^2 condition.

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1. Introduction

Let (M, g) be a compact Riemann surface, in this article we consider the following $SU(n+1)$ Toda system defined on M:

$$\Delta_g v_i + \sum_{j=1}^{n} a_{ij} H_j e^{v_j} - K(x) = 4\pi \sum_m \gamma_{mi} \delta_{q_m}, \quad 1 \le i \le n \tag{1.1}$$

where Δ_g is the Laplace–Beltrami operator $(-\Delta_g \ge 0)$, $H_1, .., H_n$ are positive smooth functions, K is the Gauss curvature, δ_{q_m} stands for the Dirac measure at q_m, $A = (a_{ij})_{n\times n}$ is the following Cartan matrix:

$$A = \begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \\ 0 & \dots & -1 & 2 & -1 \\ 0 & \dots & & -1 & 2 \end{pmatrix}.$$

The $SU(n+1)$ Toda system is well known to have close ties with many fields in Physics and Geometry. In Geometry the solutions of the Toda system are closely related to holomorphic curves (or harmonic sequences) of M into $\mathbb{CP}^n$. In the special case $M = \mathbb{S}^2$, the space of holomorphic curves of $\mathbb{S}^2$ to $\mathbb{CP}^n$ is identical to the space of solutions to the $SU(n+1)$ system. The q_ms on the right hand side of (1.1) are ramification points of the corresponding holomorphic curve and γ_{im} is the total ramificated index at q_i. See [3,6,10,11,16] and the reference therein for discussions in detail.

On other hand in Physics, the analytic aspects of (1.1) are crucial for the understanding of the relativistic version of the non-abelian Chern–Simons models, see [1,2,4,5,7,8, 13,14,23,27,29–31] and the reference therein.

Using the Green's function

$$\begin{cases} -\Delta_g G(y, \cdot) = \delta_p - 1, \\ \int_M G(y, \eta) dV_g(\eta) = 0 \end{cases} \tag{1.2}$$

and a standard transformation (see [22]) we can eliminate the singularity on the right hand side of (1.1) and rewrite (1.1) as

$$\Delta_g u_i + \sum_{j=1}^{n} a_{ij} \rho_j \left(\frac{h_j e^{u_j}}{\int_M h_j e^{u_j} dV_g} - 1 \right) = 0, \quad \text{in} \quad M, \quad i = 1, .., n \tag{1.3}$$

where $\rho_i > 0$ are constants, $h_1, .., h_n$ are nonnegative continuous functions on M, and for convenience, we assume the volume of M is 1. It is easy to see that if $u = (u_1, .., u_n)$

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