Advances in Mathematics 285 (2015) 1415–1447

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

A transchromatic proof of Strickland's theorem

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ARTICLE INFO

Article history:
Received 27 June 2014
Received in revised form 23 July 2015
Accepted 31 July 2015
Available online 9 September 2015
Communicated by Mark Behrens

Keywords:
Morava E-theory
Chromatic homotopy
Character theory

ABSTRACT

In [15] Strickland proved that the Morava E-theory of the symmetric group has an algebro-geometric interpretation after taking the quotient by a certain transfer ideal. This result has influenced most of the work on power operations in Morava E-theory and provides an important calculational tool. In this paper we give a new proof of this result as well as a generalization by using transchromatic character theory. The character maps are used to reduce Strickland's result to representation theory.

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1. Introduction and outline

The coefficient ring of Morava E-theory carries the universal deformation of a height n formal group over a perfect field of characteristic p. This formal group seems to determine the Morava E-theory of a large class of spaces. An example of this is the important result of Strickland's [15] that describes the E-theory of the symmetric group (modulo a transfer ideal) as the scheme that classifies subgroups in the universal deformation. This result plays a critical role in the study of power operations for Morava E-theory [9–11] and explicit calculations of the E-theory of symmetric groups [8,18] and the spaces $L(k)$ [4].

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http://dx.doi.org/10.1016/j.aim.2015.07.025
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We exploit a method that reduces facts such as the existence of Strickland's isomorphism into questions in representation theory by using the transchromatic generalized character maps of [13]. In this paper we illustrate the method by giving a new proof of Strickland's result as well as a generalization to wreath products of abelian groups with symmetric groups. The new feature here is more than the generalization of Strickland's result to certain p-divisible groups, it is a method for reducing a class of hard problems in E-theory to representation theory.

We explain some of the ideas. Let G be a finite group. There is an endofunctor of finite G-CW complexes $\mathcal{L}$ called the (p-adic) inertia groupoid functor that has some very useful properties:

- Given a cohomology theory E_G on finite G-CW complexes, the composite $E_G(\mathcal{L}(-))$ is a cohomology theory on finite G-CW complexes.
- Let $*$ be a point with a G-action. There is an equivalence

$$EG \times_G \mathcal{L}(*) \simeq \mathrm{Map}(B\mathbb{Z}_p, BG).$$

  The right hand side is the (p-adic) free loop space of BG.
- If E is p-complete, characteristic 0, and complex oriented with formal group $\mathbb{G}_E$ then (working Borel equivariantly) the isomorphisms

$$E^0_{\mathbb{Z}/p^k}(\mathcal{L}(*)) \cong E^0(\coprod_{\mathbb{Z}/p^k} B\mathbb{Z}/p^k) \cong \prod_{\mathbb{Z}/p^k} E^0(B\mathbb{Z}/p^k)$$

  imply that, as k varies, the algebro-geometric object associated to $E_{\mathbb{Z}/p^k}(\mathcal{L}(-))$ is the p-divisible group $\mathbb{G}_E \oplus \mathbb{Q}_p/\mathbb{Z}_p$.
- The target of the character maps of [6] and [13] take values in a cohomology theory built using $\mathcal{L}$.

Because of the second property we feel safe abusing notation and writing $EG \times_G \mathcal{L}(*)$ and $\mathcal{L}BG$ interchangeably. The latter is certainly easier on the eyes.

Now let E be Morava E_n. The p-divisible group associated to $\mathbb{G}_E$ is the directed system built out of the p^k-torsion as k varies

$$\mathbb{G}_E[p] \to \mathbb{G}_E[p^2] \to \dots .$$

We will be interested in finite subgroups of $\mathbb{G}_E$ and related p-divisible groups. A subgroup will always mean a finite flat subgroup scheme of constant rank (order). Given such a finite flat subgroup scheme H we will denote its order by $|H|$.

Precomposing with the inertia groupoid h times gives a cohomology theory $E(\mathcal{L}^h(-))$ with associated p-divisible group $\mathbb{G}_E \oplus \mathbb{Q}_p/\mathbb{Z}_p^h$, where $\mathbb{Q}_p/\mathbb{Z}_p^h = (\mathbb{Q}_p/\mathbb{Z}_p)^h$. In [6] and [13] rings called C_t for $0 \leq t < n$ are constructed with three important properties:

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