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Cyclic Hodge integrals and loop Schur functions

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ABSTRACT

We conjecture an evaluation of three-partition cyclic Hodge integrals in terms of loop Schur functions. Our formula implies the orbifold Gromov–Witten/Donaldson–Thomas correspondence for toric Calabi–Yau threefolds with transverse A_n singularities. We prove the formula in the case where one of the partitions is empty, and thus establish the orbifold Gromov–Witten/Donaldson–Thomas correspondence for local toric surfaces with transverse A_n singularities.

1. Introduction

1.1. Statement of results

This paper investigates the relationship between the Gromov–Witten theory and the Donaldson–Thomas theory of Calabi–Yau 3-orbifolds with transverse A_n singularities.

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The Gromov–Witten partition function $GW^{\bullet}(\mathcal{X};\mathbf{x},u,v)$ is a formal series encoding intersection numbers on the moduli stack of orbifold stable maps to $\mathcal{X}$ and the Donaldson–Thomas partition function $DT'(\mathcal{X};\mathbf{q},v)$ is a formal series encoding intersection numbers on the Hilbert scheme of substacks in $\mathcal{X}$. We study the following conjecture, first suggested by Bryan, Cadman and Young [3].

Conjecture 1.1. *After the change of variables $\mathbf{q}(\mathbf{x},u)$ specified in Section 1.2.4, we have*

$$DT'(\mathcal{X};\mathbf{q},v) = GW^{\bullet}(\mathcal{X};\mathbf{x},u,v).$$

The variables $\mathbf{q}$ and $\mathbf{x}$ are subdivided into sets of variables corresponding to each singular curve in $\mathcal{X}$; in Section 1.2.4 we describe the precise change of variables between these sets of variables.

In the toric case, both the GW and DT partition functions can be decomposed into contributions defined locally at each torus fixed point [3,16], the so-called orbifold topological vertex. One motivation for the topological vertex is to use this decomposition to reduce global phenomena to the local setting. We carry out such a reduction explicitly for Conjecture 1.1.

Theorem 1.1 *(Theorem 2.1). In the toric setting, Conjecture 1.1 is implied by an explicit correspondence between the contributions at each torus fixed point.*

In [16], the local GW partition function at each torus fixed point was computed explicitly in terms of three-partition cyclic Hodge integrals on the moduli stack of stable maps into the classifying stacks $\mathcal{B}\mathbb{Z}_n$, where n is a positive integer. These local contributions are indexed by triples of conjugacy classes (μ^1,μ^2,μ^3) in the generalized symmetric groups $\mathbb{Z}_n \wr S_{|\mu^i|}$. Here

$$\mathbb{Z}_n \wr S_{|\mu^i|} = \{(k,\sigma) | k=(k_1,\cdots,k_{|\mu^i|}), \sigma \in S_{|\mu^i|}\}$$

with multiplication defined by

$$(k,\sigma)\cdot(k',\sigma') = (k\sigma(k'),\sigma\sigma').$$

In [3], the local DT contributions at each torus fixed point were computed in closed form in terms of loop Schur functions. The local DT contributions are indexed by triples of irreducible representations $(\sigma^1,\sigma^2,\sigma^3)$ in $\mathbb{Z}_n \wr S_{|\sigma^i|}$. The local GW/DT correspondence of Theorem 2.1 relates the local partition functions through a change of variables and by identifying the indexing triples via the character table of $\mathbb{Z}_n \wr S_d$. The main result of the current paper is the following.

Theorem 1.2 *(Theorem 2.2). After the change of variables $\mathbf{q}(\mathbf{x},u)$ specified in Section 1.2.4, the local GW/DT correspondence holds whenever $\mu^i = \sigma^i = \emptyset$ for at least one i.*