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Independent random partial matching with general types

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A B S T R A C T

We provide a mathematical foundation for an independent random *partial* matching with a continuum of agents, where the type space is not necessarily *finite*. Besides establishing the existence result, we also identify the deterministic distributions of matched types in the matching. Our results are based on the Fubini property for Loeb product spaces and products of Loeb transition probabilities, as well as Keisler's homogeneity theorem for Loeb counting probability spaces.

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1. Introduction

The mathematical framework of random matchings with a continuum of agents has now been widely used in biology, economics, finance and many other fields.[1] Furthermore, in applications, a matching is often a partial matching where there are some unmatched agents; see [20] for example. Mathematically, if one takes the usual measure-theoretic product of a given space of agents and a given sample space in a random matching, then the product should have the Fubini property in the sense that the integral of a function on the product is the same as the iterated integrals. As noted by Keisler (*e.g.*, [10,18]), the Loeb product space that extends the product of two Loeb spaces retains the Fubini property. Later, to address an independent random *partial* matching with a continuum of agents and *finite* types, Duffie and Sun [3] showed the Fubini property holds for finite or infinite products of Loeb transition probabilities. However, the finiteness assumption on types is not sufficient to adequately represent many situations in matching; see [20] among others.[2] A mathematical foundation for a *partial* matching with a general *type* space—being either finite or infinite—is still needed for an independent random matching with a continuum of agents. This motivates the paper.

The tools in [3] are not enough to address such a partial matching with general types since their proofs rely on the finiteness of the type space. In order to fill the gap, we need to go beyond the use of the Fubini property for products of Loeb transition probabilities. A key result that we rely on is Keisler's homogeneity theorem for Loeb counting probability spaces as in [11,13].[3] Recall that a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ is said to be homogeneous if there always exists a probabilistic automorphism between any two given random variables x and y on Ω with the same distribution. Apparently, the homogeneity property holds when Ω is finite, and fails to hold for the usual Lebesgue unit interval. Moreover, it is now well known that every Loeb counting probability space is homogeneous; see [11,13]. These observations are directly related to the existence re-

[1] See [2,8] for example. See also [3,4] for extensive references on biology, economics, mathematics and finance.

[2] In [20], the type space is $[0, \infty)$. For independent random full matchings with a general type space, [3] and [4] established the mathematical foundation; see [22] for an alternative proof on the existence.

[3] The homogeneity is used in the nonstandard approach to stochastic analysis and mathematical economics; see [12,14]. For recent developments and applications of nonstandard analysis, see [1,5,13,16,17,21,25].