

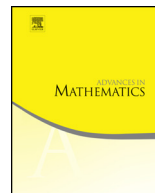


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Cone-volume measure of general centered convex bodies



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ABSTRACT

We show that the cone-volume measure of a convex body with centroid at the origin satisfies the subspace concentration condition. This extends former results obtained in the discrete as well as in the symmetric case and implies, among others, a conjectured best possible inequality for the U-functional of a convex body.

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1. Introduction

Let \mathcal{K}^n be the set of all convex bodies in \mathbb{R}^n having non-empty interiors, i.e., $K \in \mathcal{K}^n$ is a convex compact subset of the n -dimensional Euclidean space \mathbb{R}^n with $\text{int}(K) \neq \emptyset$. As usual, we denote by $\langle \cdot, \cdot \rangle$ the inner product on $\mathbb{R}^n \times \mathbb{R}^n$ with associated Euclidean norm $\| \cdot \|$, and $S^{n-1} \subset \mathbb{R}^n$ denotes the $(n-1)$ -dimensional unit sphere, i.e., $S^{n-1} = \{x \in \mathbb{R}^n : \|x\| = 1\}$.

For $K \in \mathcal{K}^n$ we write $S_K(\cdot)$ and $h_K(\cdot)$ to denote its surface area measure and support function, respectively, and ν_K to denote the Gauß map assigning the outer unit normal $\nu_K(x)$ to an $x \in \partial_* K$, where $\partial_* K$ consists of all points in the boundary ∂K of K having a unique outer normal vector. If the origin o lies in $K \in \mathcal{K}^n$, the *cone-volume measure* of K on S^{n-1} is given by

$$V_K(\omega) = \int_{\omega} \frac{h_K(u)}{n} dS_K(u) = \int_{\nu_K^{-1}(\omega)} \frac{\langle x, \nu_K(x) \rangle}{n} d\mathcal{H}_{n-1}(x), \quad (1.1)$$

where $\omega \subseteq S^{n-1}$ is a Borel set and, in general, $\mathcal{H}_k(x)$ denotes the k -dimensional Hausdorff measure. Instead of $\mathcal{H}_n(\cdot)$, we also write $V(\cdot)$ for the n -dimensional volume.

The name cone-volume measure stems from the fact that if K is a polytope with facets F_1, \dots, F_m and corresponding outer unit normals u_1, \dots, u_m , then

$$V_K(\omega) = \sum_{i=1}^m V([o, F_i]) \delta_{u_i}(\omega). \quad (1.2)$$

Here δ_{u_i} is the Dirac delta measure on S^{n-1} concentrated at u_i , and for $x_1, \dots, x_m \in \mathbb{R}^n$ and subsets $S_1, \dots, S_l \subseteq \mathbb{R}^n$ we denote the convex hull of the set $\{x_1, \dots, x_m, S_1, \dots, S_l\}$ by $[x_1, \dots, x_m, S_1, \dots, S_l]$. With this notation $[o, F_i]$ is the cone with apex o and basis F_i .

In recent years, cone-volume measures have appeared and were studied in various contexts, see, e.g., F. Barthe, O. Guédon, S. Mendelson and A. Naor [6], K.J. Böröczky, E. Lutwak, D. Yang and G. Zhang [10,11], M. Gromov and V.D. Milman [18], M. Ludwig [28], M. Ludwig and M. Reitzner [29], E. Lutwak, D. Yang and G. Zhang [32], A. Naor [34], A. Naor and D. Romik [35], G. Paouris and E. Werner [36], A. Stancu [42], G. Zhu [45,46], K.J. Böröczky and P. Hegedüs [8].

In particular, cone-volume measures are the subject of the *logarithmic Minkowski problem*, which is the particular interesting limiting case $p = 0$ of the general L_p -Minkowski problem – one of the central problems in convex geometric analysis. It is the task:

Find necessary and sufficient conditions for a finite Borel measure μ on S^{n-1} to be the cone-volume measure V_K of $K \in \mathcal{K}^n$ with o in its interior.

In the recent paper [11], K.J. Böröczky, E. Lutwak, D. Yang and G. Zhang solved the logarithmic Minkowski problem in the even case, i.e., they characterized the cone-volume

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