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The discriminant criterion and automorphism groups of quantized algebras

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We compute the automorphism groups of some quantized algebras, including tensor products of quantum Weyl algebras and some skew polynomial rings.

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0. Introduction

It is well known that every automorphism of the polynomial ring $k[x]$, where k is a field, is determined by the assignment $x \mapsto ax + b$ for some $a \in k^\times := k \setminus \{0\}$ and $b \in k$. Every automorphism of $k[x_1, x_2]$ is *tame*, that is, it is generated by affine and elementary automorphisms (defined below). This result was first proved by Jung [10] in 1942 for characteristic zero and then by van der Kulk [20] in 1953 for arbitrary characteristic. A structure theorem for the automorphism group of $k[x_1, x_2]$ was also given in [20]. The automorphism group of $k[x_1, x_2, x_3]$ has not yet been fully understood, and the best result in this direction is the existence of wild automorphisms (e.g. the Nagata automorphism) by Shestakov–Umirbaev [16].

The automorphism group of the skew polynomial ring $k_q[x_1, \dots, x_n]$, where $q \in k^\times$ is not a root of unity and $n \geq 2$, was completely described by Alev and Chamarie [2, Theorem 1.4.6] in 1992. Since then, many researchers have been successfully computing the automorphism groups of classes of interesting infinite-dimensional noncommutative algebras, including certain quantum groups, generalized quantum Weyl algebras, skew polynomial rings and many more – see [2–5,8,19,21,22], among others. In particular, Yakimov has proved the Andruskiewitsch–Dumas conjecture and the Launois–Lenagan conjecture by using a rigidity theorem for quantum tori, see [21,22], each of which determines the automorphism group of a family of quantized algebras with parameter q being not a root of unity. See also [9] for a uniform approach to these two conjectures.

Determining the automorphism group of an algebra is generally a very difficult problem. In [6] we introduced the discriminant method to compute automorphism groups of some noncommutative algebras. In this paper we continue to develop new methods and extend ideas from [6] for both discriminants and automorphism groups.

Suppose A is a filtered algebra with filtration $\{F_i A\}_{i \geq 0}$ such that the associated graded algebra $\operatorname{gr} A$ is generated in degree 1. An automorphism g of A is *affine* if $g(F_1 A) \subset F_1 A$. An automorphism h of the polynomial extension $A[t]$ is called *triangular* if there is a $g \in \operatorname{Aut}(A)$, $c \in k^\times$ and r in the center of A such that

$$h(t) = ct + r \quad \text{and} \quad h(x) = g(x) \in A \quad \text{for all } x \in A.$$

As in [6], we use the discriminant to control automorphisms and locally nilpotent derivations. Let $C(A)$ denote the center of A. Here is the discriminant criterion for affine automorphisms.

Theorem 1. *Assume k is a field of characteristic 0. Let A be a filtered algebra, finite over its center, such that the associated graded ring $\operatorname{gr} A$ is a connected graded domain. Suppose that the v-discriminant $d_v(A/C(A))$ is dominating for some $v \geq 1$. Then the following hold.*

(1) *Every automorphism of A is affine, and $\operatorname{Aut}(A)$ is an algebraic group that fits into the exact sequence*