

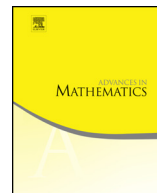


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Suffridge's convolution theorem for polynomials and entire functions having only real zeros



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ABSTRACT

We present a q -extension of Pólya's and Schur's characterization of multiplier sequences and a q -extension and a converse of Newton's inequalities.

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1. Introduction

In [24] Rota states: “Grace’s theorem is an instance of what might be called a sturdy theorem. For almost one hundred years it has resisted all attempts at generalization. Almost all known results about the distribution of zeros of polynomials in the complex plane are corollaries of Grace’s theorem.”

The following equivalent formulation of Grace’s theorem is due to Szegő.

Theorem 1. (See Grace [11], Szegő [34].) Let

$$F(z) = \sum_{k=0}^n \binom{n}{k} a_k z^k \quad \text{and} \quad G(z) = \sum_{k=0}^n \binom{n}{k} b_k z^k$$

be polynomials of degree $n \in \mathbb{N}$ and suppose $K \subset \mathbb{C}$ is an open or closed disk or half-plane, or the open or closed exterior of a disk, that contains all zeros of F . If $G(0) \neq 0$, then each zero γ of

$$F *_{GS} G(z) := \sum_{k=0}^n \binom{n}{k} a_k b_k z^k$$

is of the form $\gamma = -\alpha\beta$ with $\alpha \in K$ and $G(\beta) = 0$. If $G(0) = 0$, then this continues to hold as long as K is not the open or closed exterior of a disk.

It is hard to disagree with Rota regarding the importance of Grace’s theorem: It includes or implies numerous other results concerning the zero location of complex polynomials and has found many applications in complex analysis and other fields. For instance, it forms the basis of the geometric convolution theory which was developed by Ruscheweyh, Suffridge, and Sheil-Small (see [25,26,30–33] and, more recently, [27–29]) and it can be used to classify all linear operators which preserve the set of polynomials whose zeros lie in a given circular domain (cf. [26, Thm. 1.1], [32, Sec. 5.8], and [2]). Very recently, in an impressive series of papers [3–5], Borcea and Brändén used Grace’s theorem in order to develop a unified analytic theory of multivariate polynomials with many astonishing applications.

One might argue, however, whether Rota was correct regarding the resistency of Grace’s theorem at generalization. In fact, in 1976 Suffridge [33] (see also [16] for a different proof) showed the following stunning extension of Grace’s theorem (without having any further information the author would guess that Rota was simply unaware of Suffridge’s result).

Theorem 2 (Suffridge). For $\lambda \in [0, \frac{2\pi}{n})$ let $\mathcal{P}_n(\lambda)$ denote the set of polynomials of degree n that have all their zeros on the unit circle $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$ with each pair of zeros separated by an angle of at least λ . Then for

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