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Different asymptotic behavior versus same dynamical complexity: Recurrence & (ir)regularity

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ABSTRACT

For any dynamical system $T : X \to X$ of a compact metric space X with g-almost product property and uniform separation property, under the assumptions that the periodic points are dense in X and the periodic measures are dense in the space of invariant measures, we distinguish various periodic-like recurrences and find that they all carry full topological entropy and so do their gap-sets. In particular, this implies that any two kind of periodic-like recurrences are essentially different. Moreover, we coordinate periodic-like recurrences with (ir)regularity and obtain lots of generalized multi-fractal analyses for all continuous observable functions. These results are suitable for all β-shifts ($\beta > 1$), topological mixing subshifts of finite type, topological mixing expanding maps or topological mixing hyperbolic diffeomorphisms, etc.
Roughly speaking, we combine many different "eyes" (i.e., observable functions and periodic-like recurrences) to observe the dynamical complexity and obtain a *Refined Dynamical Structure* for Recurrence Theory and Multi-fractal Analysis.

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1. Introduction

In the theory of dynamical systems, i.e., the study of the asymptotic behavior of orbits $\{T^n(x)\}_{n\in\mathbb{N}}$ (denoted by $Orb(x)$) when $T : X \to X$ is a continuous map of a compact metric space X and $x \in X$, one may say that two fundamental problems are to understand how to partition different asymptotic behavior and how the points with same asymptotic behavior control or determine the complexity of system T.

Topological entropy is a classical concept to describe the dynamical complexity. In this paper we are mainly to deal with a certain class of dynamical systems and show that various subsets characterized by distinct asymptotic behavior all carry full topological entropy. To make this more precise let us introduce the following terminology. $T : X \to X$ is a continuous map of a compact metric space X.

Definition 1.1. For a collection of subsets $Z_1, Z_2, \cdots, Z_k \subseteq X$ $(k \geq 2)$, we say $\{Z_i\}$ has *full entropy gaps* with respect to $Y \subseteq X$ if

$$h_{top}(T, (Z_{i+1} \setminus Z_i) \cap Y) = h_{top}(T, Y) \quad \text{for all } 1 \leq i < k,$$

where $h_{top}(T, Z)$ denotes the topological entropy of a set $Z \subseteq X$.

Often, but not always, the sets Z_i are nested $(Z_i \subseteq Z_{i+1})$. Remark that for any system with zero topological entropy, it is obvious that any collection $\{Z_i\}$ has full entropy gaps with respect to any $Y \subseteq X$. Notice that if X is a finite set, then any system on X is simple and carries zero entropy. Thus in present paper, we always assume that

X is a compact metric space with infinitely many points.

Given $x \in M$, let $\omega_T(x)$ denote the ω-limit set of x, let $M_x(T)$ be the limit set of the empirical measures for x and let $C_x := \overline{\bigcup_{\nu \in M_x(T)} S_\nu}$ where S_ν denotes the support of measure ν. In this paper, we consider the following subsets of X according to different asymptotic behavior:

$$\begin{aligned}
Per(T) &:= \{\text{periodic points of } T\},\\
A(T) &:= \{\text{almost periodic points of } T\} = \{\text{ points contained in minimal set}\},\\
Rec(T) &:= \{\text{recurrent points of } T\},\\
\Omega(T) &:= \{\text{non-wandering points of } T\},\\
W(T) &:= \{x \in Rec(T) \mid S_\mu = C_x \text{ for every } \mu \in M_x(T)\},\\
QW(T) &:= \{x \in Rec(T) \mid C_x = \omega_T(x)\},\\
V(T) &:= \{x \in QW(T) \mid \exists\, \mu \in M_x(T) \text{ such that } S_\mu = C_x\}.
\end{aligned}$$