Advances in Mathematics 288 (2016) 576–630

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Linked symplectic forms and limit linear series in rank 2 with special determinant

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ARTICLE INFO

Article history:
Received 7 November 2014
Received in revised form 3 August 2015
Accepted 8 September 2015
Available online xxxx
Communicated by Ravi Vakil

Keywords:
Vector bundles
Limit linear series
Symplectic forms
Brill–Noether theory
Coherent systems

ABSTRACT

We generalize the prior linked symplectic Grassmannian construction, applying it to prove smoothing results for rank-2 limit linear series with fixed special determinant on chains of curves. We apply this general machinery to prove new results on nonemptiness and dimension of rank-2 Brill–Noether loci in a range of degrees.

Contents

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[1] The first author was partially supported by NSA grant H98230-11-1-0159 and Simons Foundation grant #279151 during the preparation of this work.

http://dx.doi.org/10.1016/j.aim.2015.09.030
0001-8708/

1. Introduction

At its most basic, higher-rank Brill–Noether theory addresses the question: for a general curve of genus g, how many global sections can a (semi)stable vector bundle of given rank and degree have? This has been an active subject of study for more than 20 years, and the picture which has emerged is one of complexity, with no comprehensive conjectures even in the case of rank 2. Nonetheless, those cases which have been understood have already been important in a number of strikingly different contexts, from Mukai's work [10] on classification of Fano threefolds to recent work of Bhosle, Brambila-Paz and Newstead [1] on Butler's conjecture. See [7] and the introduction of [15] for a more detailed survey.

In the present paper, we consider $\mathfrak{g}^k_{2,d}$s consisting of pairs $(\mathscr{E}, V)$, where $\mathscr{E}$ is a vector bundle of rank 2 and degree d, and V is a k-dimensional space of global sections. We will be interested in producing such pairs where the underlying bundles are stable or semistable; under different stability conditions which also take into account the space of sections, such pairs are also often called 'coherent systems.' We prove new results on existence of $\mathfrak{g}^k_{2,d}$s (occurring in families of the suitably modified expected dimension) in a wide range of degrees and genera. Note that in contrast to the classical case, in the higher-rank case existence results appear to be at least as difficult as Gieseker–Petri-type theorems. Our main tool is a new smoothing theorem in the case of special determinant, and as in the classical rank-1 case treated by Eisenbud and Harris, our smoothing theorem is inseparable from a careful study of dimension. Now, the classical expected dimension ρ generalizes to higher rank, and in some cases gives the correct dimension for the moduli stack of $\mathfrak{g}^k_{r,d}$s, but is not enough to understand the general picture. One case where the picture seems clearer is that of rank-2 vector bundles with fixed canonical determinant: Bertram, Feinberg [2] and Mukai [9] observed that in this case, the behavior appears to be closer to that of the classical rank-1 case, albeit with a modified expected dimension. While the naive expected dimension for the fixed determinant case is $\rho - g$, they showed that symmetries in the canonical determinant case forced the dimension to be at least $\rho_\omega := \rho - g + \binom{k}{2}$, and they conjectured that this is in fact the correct dimension, and in particular that the relevant moduli stacks should be nonempty when $\rho_\omega \geq 0$. The existence portion of their conjecture remains open, while their results on modified expected dimension were generalized by the first author to a wider class of special determinants in [13].