Advances in Mathematics 288 (2016) 825–886

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Bridgeland stability conditions on the acyclic triangular quiver

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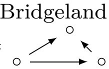
A R T I C L E I N F O

Article history:
Received 23 October 2014
Received in revised form 20 October 2015
Accepted 26 October 2015
Available online 21 November 2015
Communicated by Tony Pantev

Keywords:
Bridgeland stability conditions on triangulated categories
Exceptional collections
Tame representation type quiver
Affine quiver
Contractible space

A B S T R A C T

Using results in our previous paper "Non-semistable exceptional objects in hereditary categories", we focus here on studying the topology of the space of Bridgeland stability conditions on $D^b(Rep_k(Q))$, where $Q =$  and k is an algebraically closed field. In particular, we prove that this space is contractible (in the previous paper it was shown that it is connected).

Contents

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http://dx.doi.org/10.1016/j.aim.2015.10.014
0001-8708/

1. Introduction

In 1994 Maxim Kontsevich interpreted a duality coming from physics in a powerful mathematical framework called Homological Mirror Symmetry (HMS). HMS is now the foundation of a wide range of mathematical research. Numerous works by many authors have demonstrated the interaction of mirror symmetry and HMS with new and subtle mathematical structures. One of these structures is "the moduli space" of stability conditions.

The study of stability in triangulated categories was initiated by M. Douglas and mathematically by T. Bridgeland. More precisely, Bridgeland defined in [3] the space of stability conditions on a triangulated category $\mathcal{T}$, denoted by $\mathrm{Stab}(\mathcal{T})$, and equipped it with a structure of a complex manifold on which act the groups $\widetilde{GL}^+(2, \mathbb{R})$ and $\mathrm{Aut}(\mathcal{T})$.

The majority of the activity since then has focused on categories of algebro-geometric origin. Significant work in this direction is due to T. Bridgeland, A. King, E. Macrì, S. Okada, Y. Toda, A. Bayer, J. Woolf, J. Collins, A. Polishchuck et al. [21,4,1,24,23].

In previous works [12,13] we developed results and ideas by T. Bridgeland [3], A. King [18], E. Macrì [20], J. Collins and A. Polishchuck [8].

A parallel between dynamical systems and categories is being established [13,5,15,19,3,4] and the study of the topology of the spaces of stability conditions became a subject of significant importance. According to this analogy the stability space plays the role of the Teichmüller space. In such a way the moduli space of stability conditions provides a link between: topology, representation theory, dynamical systems, category theory.

Recently J. Woolf in [25] and N. Broomhead, D. Pauksztello, D. Ploog in [7] showed classes of categories with contractible components in the space of stability conditions. These papers generalize and unify various known results (e.g. results in [6,22]) for stability spaces of specific categories, and settle some conjectures about the stability spaces associated to Dynkin quivers, and to their Calabi–Yau–N Ginzburg algebras (the latter are not in the scope of [7]). However the results in [25,7] do not cover tame representation type quivers.

In the present paper we give a new example of a tame representation type quiver with contractible space of stability conditions. This paper is a natural consequence of our