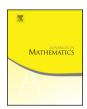


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## Advances in Mathematics





# On quantizations of complex contact manifolds



### Pietro Polesello<sup>1</sup>

 $Dipartimento\ di\ Matematica,\ Universit\`a\ di\ Padova,\ via\ Trieste\ 63,\ 35121\ Padova,\ Italy$ 

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#### ABSTRACT

A (holomorphic) quantization of a complex contact manifold is a filtered algebroid stack which is locally equivalent to the ring  $\mathcal E$  of microdifferential operators and which has trivial graded. The existence of a canonical quantization has been proved by Kashiwara. In this paper we consider the classification problem, showing that the above quantizations are classified by the first cohomology group with values in a certain sheaf of homogeneous forms. Secondly, we consider the problem of existence and classification for quantizations given by algebras.

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#### 0. Introduction

This paper deals with the problem of existence and classification for the quantizations of complex contact manifolds.

E-mail address: pietro@math.unipd.it.

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Precisely, consider a complex contact manifold  $(Y, \alpha)$ , *i.e.* a complex manifold Y endowed with a projective 1-form  $\alpha$  (a global section of the projective cotangent bundle  $P^*Y \to Y$ ) such that  $d\alpha|_H$  is non-degenerate, where H denotes the codimension 1 subbundle of the tangent bundle associated to  $\alpha$  via projective duality. Let  $\mathcal{L} = \alpha^*\mathcal{O}_{P^*Y}(1)$  be the pull-back of the relative Serre sheaf on  $P^*Y$ : it is a locally free  $\mathcal{O}_Y$ -module of rank 1 endowed with a Lie bracket induced by  $\alpha$  (the Lagrange bracket). Finally, denote by  $\mathcal{S}_Y$  the sheaf on Y whose local sections are symbols, *i.e.* series  $\sum_{j=-\infty}^m f_j$  with  $f_j \in \mathcal{L}^{\otimes j}$  and satisfying some growth conditions.  $\mathcal{S}_Y$  is endowed with a natural filtration and the associated graded sheaf is the "homogeneous coordinates ring"  $\mathcal{O}_Y^{hom} = \bigoplus_{m \in \mathbb{Z}} \mathcal{L}^{\otimes m}$ .

A (holomorphic) quantization algebra on  $(Y, \alpha)$  is a sheaf of algebras  $\mathcal{A}$  on Y locally isomorphic to  $\mathcal{S}_Y$  as  $\mathbb{C}_Y$ -modules, in such a way that the product on  $\mathcal{S}_Y$  induced by that of  $\mathcal{A}$  is a formal bidifferential operator and is compatible both with the algebra structure on  $\mathcal{O}_Y$  and with the Lie bracket on  $\mathcal{L}$ . Such  $\mathcal{A}$  is naturally filtered with  $\mathcal{O}_Y^{hom}$  as graded sheaf and the above compatibility conditions mean that  $\operatorname{Gr}(\mathcal{A}) \simeq \mathcal{O}_Y^{hom}$  is a graded algebra isomorphism commuting with the Poisson structures (induced by the commutator on the left-hand side and by the Lagrange bracket on the right-hand side).

Darboux's theorem for complex contact manifolds asserts that for any point  $y \in Y$  there exists an open neighborhood V of y and a contact transformation  $i: V \to P^*M$  for some complex manifold M (here  $P^*M$  is endowed with the canonical contact structure given by the Liouville 1-form). Since any quantization algebra on V is locally isomorphic through i to the  $\mathbb{C}$ -algebra  $\mathcal{E}_{P^*M}$  of Sato's microdifferential operators (a localization of the algebra  $\mathcal{D}_M$  of differential operators on M), it follows that the quantization algebras on Y are nothing but  $\mathcal{E}$ -algebras, i.e.  $\mathbb{C}$ -algebras locally isomorphic to  $i^{-1}\mathcal{E}_{P^*M}$  for any Darboux chart  $i: Y \supset V \to P^*M$ .

In the case  $Y = P^*M$ , formal (*i.e.* without growth conditions)  $\mathcal{E}$ -algebras were classified by Boutet de Monvel in [3].

For a complex contact manifold Y, the situation is more complicated, since  $\mathcal{E}$ -algebras may not exist globally. However Kashiwara in [19] proved that there exists a canonical stack (sheaf of categories) of modules over locally defined  $\mathcal{E}$ -algebras. In fact, this stack is equivalent to the stack of modules over an algebroid stack  $\mathsf{E}_Y$  (see [23,9]). Moreover,  $\mathsf{E}_Y$  has the same properties of an  $\mathcal{E}$ -algebra: it is filtered, locally equivalent to an  $\mathcal{E}$ -algebra and it has  $\mathcal{O}_V^{hom}$  as associated graded stack.

Hence, it makes sense to say that  $\mathsf{E}_Y$  is a (holomorphic) quantization of Y and to replace  $\mathcal{E}$ -algebras by  $\mathbb{C}$ -linear stacks locally equivalent to  $\mathsf{E}_Y$ . Being locally modeled on  $\mathcal{E}$ -algebras, these objects have locally trivial graded, but their associated graded stack in general is non-trivial. Thus, we are lead to define a  $(\mathcal{E}, \sigma)$ -algebroid, playing the role of quantization of Y, as a filtered  $\mathbb{C}$ -linear stacks which is locally equivalent to  $\mathsf{E}_Y$  and which has  $\mathcal{O}_Y^{hom}$  as associated graded stack.

In this paper we classify all  $(\mathcal{E}, \sigma)$ -algebroids on any complex contact manifold Y. This is done by means of the cohomology group  $H^1(Y; \Omega_Y^{1,cl}(0))$ , where  $\Omega_Y^{1,cl}(0)$  denotes the push-forward of the sheaf of closed 0-homogeneous 1-forms on the canonical

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