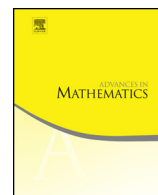




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Holonomy transformations for singular foliations

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ABSTRACT

In order to understand the linearization problem around a leaf of a singular foliation, we extend the familiar holonomy map from the case of regular foliations to the case of singular foliations. To this aim we introduce the notion of holonomy transformation. Unlike the regular case, holonomy transformations cannot be attached to classes of paths in the foliation, but rather to elements of the holonomy groupoid of the singular foliation.

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0. Introduction

0.1. Historical overview and motivations

A great deal of foliation theory is based on the understanding of the action of the holonomy pseudogroup on the transversal structure of the foliation. Geometrically, the holonomy of a (regular) foliation (M, F) at a point $x \in M$ is realized by a map $h_x : \pi_1(L) \rightarrow \text{GermDiff}(S)$, where L is the leaf at x , S is a transversal at x , and $\text{GermDiff}(S)$ is the space of germs of local diffeomorphisms of S . Its linearization $\text{Lin}(h_x) : \pi_1(L) \rightarrow GL(N_x L)$ is a representation on the normal space to L at x . When one considers all pairs of points in leaves of M , the linearization gives rise to a representation $\text{Lin}(h)$ of the holonomy groupoid on TM/F , the normal bundle to the leaves. Notice that TM/F plays the role of the tangent bundle of the quotient space M/F (cf. [7, §10.2]), and is

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