Advances in Mathematics 256 (2014) 348–397

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Holonomy transformations for singular foliations

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ARTICLE INFO

Article history:
Received 8 June 2012
Accepted 6 February 2014
Available online 4 March 2014
Communicated by
Tomasz S. Mrowka

MSC:
primary 53C12, 53C29
secondary 22A25, 93B18

Keywords:
Singular foliation
Holonomy
Proper Lie groupoid
Linearization
Deformation

ABSTRACT

In order to understand the linearization problem around a leaf of a singular foliation, we extend the familiar holonomy map from the case of regular foliations to the case of singular foliations. To this aim we introduce the notion of holonomy transformation. Unlike the regular case, holonomy transformations cannot be attached to classes of paths in the foliation, but rather to elements of the holonomy groupoid of the singular foliation.

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[1] I. Androulidakis was partially supported by a Marie Curie Career Integration Grant (FP7-PEOPLE-2011-CIG, Grant No. PCI09-GA-2011-290823), by DFG (Germany) through project DFG-Az:Me 3248/1-1 and by FCT (Portugal) with European Regional Development Fund (COMPETE) and national funds through the project PTDC/MAT/098770/2008. He also acknowledges the support of a Severo Ochoa grant during a visit to ICMAT.

[2] M. Zambon was partially supported by CMUP and FCT (Portugal) through the programs POCTI, POSI and Ciencia 2007, by projects PTDC/MAT/098770/2008 and PTDC/MAT/099880/2008 (Portugal) and by projects MICINN RYC-2009-04065, MTM2009-08166-E, MTM2011-22612 and MINECO: ICMAT Severo Ochoa project SEV-2011-0087 (Spain). He thanks Universiteit Utrecht and Universität Göttingen for hospitality, and Daniel Peralta-Salas for advice and references.

http://dx.doi.org/10.1016/j.aim.2014.02.003

Contents

0. Introduction

0.1. Historical overview and motivations

A great deal of foliation theory is based on the understanding of the action of the holonomy pseudogroup on the transversal structure of the foliation. Geometrically, the holonomy of a (regular) foliation (M, F) at a point $x \in M$ is realized by a map $h_x : \pi_1(L) \to \mathit{GermDiff}(S)$, where L is the leaf at x, S is a transversal at x, and $\mathit{GermDiff}(S)$ is the space of germs of local diffeomorphisms of S. Its linearization $\mathit{Lin}(h_x) : \pi_1(L) \to GL(N_x L)$ is a representation on the normal space to L at x. When one considers all pairs of points in leaves of M, the linearization gives rise to a representation $\mathit{Lin}(h)$ of the holonomy groupoid on TM/F, the normal bundle to the leaves. Notice that TM/F plays the role of the tangent bundle of the quotient space M/F (cf. [7, §10.2]), and is