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Champagne subdomains with unavoidable bubbles

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Abstract

A champagne subdomain of a connected open set $U \neq \emptyset$ in $\mathbb{R}^d$, $d \geq 2$, is obtained by omitting pairwise disjoint closed balls $\overline{B}(x, r_x)$, $x \in X$, the bubbles, where X is an infinite, locally finite set in U. The union A of these balls may be unavoidable, that is, Brownian motion, starting in $U \setminus A$ and killed when leaving U, may hit A almost surely or, equivalently, A may have harmonic measure 1 for $U \setminus A$.

Recent publications by Gardiner and Ghergu ($d \geq 3$) and by Pres ($d = 2$) give rather sharp answers to the question of how small such a set A may be, when U is the unit ball.

In this paper, using a totally different approach, optimal results are obtained, which hold also for arbitrary connected open sets U.
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1. Introduction and the main results

Throughout this paper let U denote a non-empty connected open set in $\mathbb{R}^d$, $d \geq 2$. Let us say that a relatively closed subset A of U is *unavoidable* if Brownian motion, starting in $U \setminus A$

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and killed when leaving U, hits A almost surely or, equivalently, if $\mu_y^{U\setminus A}(A) = 1$, for every $y \in U \setminus A$, where $\mu_y^{U\setminus A}$ denotes the harmonic measure at y with respect to $U \setminus A$ (we note that $\mu_y^{U\setminus A}$ may fail to be a probability measure, if $U \setminus A$ is not bounded).

For $x \in \mathbb{R}^d$ and $r > 0$, let $B(x, r)$ denote the open ball of center x and radius r. Suppose that X is a countable set in U having no accumulation point in U, and let $r_x > 0$, $x \in X$, such that the closed balls $\overline{B}(x, r_x)$, the *bubbles*, are pairwise disjoint, $\sup_{x\in X} r_x/\mathrm{dist}(x, \partial U) < 1$ and, if U is unbounded, $r_x \to 0$ as $x \to \infty$. Then the union A of all $\overline{B}(x, r_x)$ is relatively closed in U, and the connected open set $U \setminus A$ (which is non-empty!) is called a *champagne subdomain of* U.

This generalizes the notions used in [3,8,12–14] for $U = B(0, 1)$; see also [6] for the case where U is $\mathbb{R}^d$, $d \geq 3$. Avoidable unions of randomly distributed balls have been discussed in [11] and, recently, in [5].

It will be convenient to introduce the set X_A for a champagne subdomain $U \setminus A$: X_A is the set of centers of all the bubbles forming A (and r_x, $x \in X_A$, is the radius of the bubble centered at x). It is fairly easy to see that, given a champagne subdomain $U \setminus A$ and a finite subset X' of X_A, the set A is unavoidable if and only if the union of all bubbles $\overline{B}(x, r_x)$, $x \in X_A \setminus X'$, is unavoidable.

The main result of Akeroyd [3] is, for a given $\delta > 0$, the existence of a champagne subdomain of the unit disc such that

$$\sum_{x\in X_A} r_x < \delta \quad \text{and yet } A \text{ is unavoidable.} \tag{1.1}$$

Ortega-Cerdà and Seip [13] improved the result of Akeroyd in characterizing a certain class of champagne subdomains $B(0, 1) \setminus A$, where A is unavoidable and $\sum_{x\in X_A} r_x < \infty$, and hence the statement of (1.1) can be obtained omitting finitely many of the discs $\overline{B}(x, r_x)$, $x \in X_A$.

Let us note that already in [10] the existence of a champagne subdomain of an arbitrary bounded connected open set U in $\mathbb{R}^2$ having property (1.1) was crucial for the construction of an example answering Littlewood's one-circle problem in the negative. In fact, Proposition 3 in [10] is a bit stronger: Even a Markov chain formed by jumps on annuli hits A before it goes to ∂U. The statement about harmonic measure (hitting by Brownian motion) is obtained by the first part of the proof of Proposition 3 in [10] (cf. also [9], where this is explicitly stated at the top of p. 72). This part uses only "one-bubble estimates" for the global Green function and the minimum principle.

Recently, Gardiner and Ghergu [8, Corollary 3] proved the following.

Theorem A. *If $d \geq 3$, then, for all $\alpha > d - 2$ and $\delta > 0$, there is a champagne subdomain $B(0, 1) \setminus A$ such that A is unavoidable and*

$$\sum_{x\in X_A} r_x^\alpha < \delta.$$

Moreover, Pres [14, Corollary 1.3] showed the following for the plane.

Theorem B. *If $d = 2$, then, for all $\alpha > 1$ and $\delta > 0$, there is a champagne subdomain $B(0, 1)\setminus A$ such that A is unavoidable and*

$$\sum_{x\in X_A} \left(\log \frac{1}{r_x}\right)^{-\alpha} < \delta.$$

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