Available online at www.sciencedirect.com

SciVerse ScienceDirect

Advances in Mathematics 244 (2013) 605–625

ADVANCES IN Mathematics

www.elsevier.com/locate/aim

Enumerative meaning of mirror maps for toric Calabi–Yau manifolds

Kwokwai Chan [a], Siu-Cheong Lau [b], Hsian-Hua Tseng [c,*]

[a] *Department of Mathematics, The Chinese University of Hong Kong, Shatin, Hong Kong*
[b] *Department of Mathematics, Harvard University, One Oxford Street, Cambridge, MA 02138, USA*
[c] *Department of Mathematics, Ohio State University, 100 Math Tower, 231 West 18th Ave., Columbus, OH 43210, USA*

Received 3 January 2012; accepted 30 May 2013
Available online 15 June 2013

Communicated by Bernd Siebert

Abstract

We prove that the inverse of a mirror map for a toric Calabi–Yau manifold of the form K_Y, where Y is a compact toric Fano manifold, can be expressed in terms of generating functions of genus 0 open Gromov–Witten invariants defined by Fukaya–Oh–Ohta–Ono (2010) [15]. Such a relation between mirror maps and disk counting invariants was first conjectured by Gross and Siebert (2011) [24, Conjecture 0.2 and Remark 5.1] as part of their program, and was later formulated in terms of Fukaya–Oh–Ohta–Ono's invariants in the toric Calabi–Yau case in Chan et al. (2012) [8, Conjecture 1.1].
© 2013 Elsevier Inc. All rights reserved.

MSC: primary 14N35; 53D45; secondary 14J33; 53D37; 53D12

Keywords: Open Gromov–Witten invariants; Mirror maps; GKZ systems; Toric manifolds; Calabi–Yau; Mirror symmetry

Contents

* Corresponding author.
E-mail addresses: kwchan@math.cuhk.edu.hk (K. Chan), s.lau@math.harvard.edu (S.-C. Lau), hhtseng@math.ohio-state.edu (H.-H. Tseng).

0001-8708/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.aim.2013.05.018

1. Introduction

Let X be an n-dimensional toric Calabi–Yau manifold, i.e. a smooth toric variety with trivial canonical line bundle $K_X \simeq \mathcal{O}_X$. Such a manifold is necessarily noncompact. Let $N = \mathbb{Z}^n$. Then $X = X_\Sigma$ is defined by a fan Σ in $N_\mathbb{R} = N \otimes_\mathbb{Z} \mathbb{R} = \mathbb{R}^n$. Let $v_0, v_1, \ldots, v_{m-1} \in N$ be the primitive generators of the 1-dimensional cones of Σ. Without loss of generality, we assume that, for $i = 0, 1, \ldots, m-1$,

$$v_i = (w_i, 1) \in N$$

for some $w_i \in \mathbb{Z}^{n-1}$ and $w_0 = 0$. Also, following Gross [22], we assume that the fan Σ has convex support so that X is a crepant resolution of an affine toric variety with Gorenstein canonical singularities.

The Picard number of X is equal to $l := m - n$. Let $\{p_1, \ldots, p_l\}$ be an nef basis of $H^2(X;\mathbb{Z})$ and let $\{\gamma_1, \ldots, \gamma_l\} \subset H_2(X;\mathbb{Z}) \cong \mathbb{Z}^l$ be the dual basis. Each 2-cycle γ_a corresponds to an integral relation

$$\sum_{i=0}^{m-1} Q_i^a v_i = 0,$$

where $Q^a := (Q_0^a, Q_1^a, \ldots, Q_{m-1}^a) \in \mathbb{Z}^m$. We equip X with a toric symplectic structure ω and regard (X, ω) as a Kähler manifold. We also complexify the Kähler class by adding a B-field $\mathbf{i}B \in H^2(X, \mathbf{i}\mathbb{R})$ and setting $\omega_\mathbb{C} = \omega + \mathbf{i}B$.

An important class of examples of toric Calabi–Yau manifolds is given by the total spaces of the canonical line bundles K_Y over compact toric Fano manifolds Y, e.g. $K_{\mathbb{P}^2} = \mathcal{O}_{\mathbb{P}^2}(-3)$.

In [8], Leung and the first two authors of this paper study local mirror symmetry for a toric Calabi–Yau manifold X from the viewpoint of the SYZ conjecture [36]. Starting with a special Lagrangian torus fibration (the Gross fibration) on X, we construct the *SYZ mirror* of X using T-duality modified by instanton corrections and wall-crossing, generalizing the constructions of Auroux [3,4]. The result is given by the following family of noncompact Calabi–Yau manifolds [8, Theorem 4.37] (see also [2, Section 7]):

$$\check{X} = \left\{ (u, v, z_1, \ldots, z_{n-1}) \in \mathbb{C}^2 \times (\mathbb{C}^\times)^{n-1} \mid uv = \sum_{i=0}^{m-1} (1 + \delta_i(q)) C_i z^{w_i} \right\}, \tag{1.1}$$

Download English Version:

https://daneshyari.com/en/article/6425763

Download Persian Version:

https://daneshyari.com/article/6425763

Daneshyari.com