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On a Morita equivalence between the duals of quantum $SU(2)$ and quantum $\widetilde{E}(2)$

Kenny De Commer [1]

Department of Mathematics, University of Tor Vergata, Via della Ricerca Scientifica 1, 00133 Rome, Italy

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Abstract

Let $SU_q(2)$ and $\widetilde{E}_q(2)$ be Woronowicz's q-deformations of respectively the compact Lie group $SU(2)$ and the non-trivial double cover of the Lie group $E(2)$ of Euclidean transformations of the plane. We prove that, in some sense, their duals are 'Morita equivalent locally compact quantum groups'. In more concrete terms, we prove that the von Neumann algebraic quantum groups $\mathscr{L}^\infty(SU_q(2))$ and $\mathscr{L}^\infty(\widetilde{E}_q(2))$ are unitary cocycle deformations of each other.

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0. Introduction

This is part of a series of papers devoted to an intriguing correspondence between the quantizations of $SU(2)$, $\widetilde{E}(2)$ and $\widetilde{SU}(1,1)$, where the latter denotes the normalizer of $SU(1,1)$ inside $SL(2,\mathbb{C})$. In a sense, their duals form a trinity of 'Morita equivalent locally compact quantum groups'. There then exists a 'linking quantum groupoid' combining these three quantum groups into one global structure, and it is important to understand for example the (co)representation theory of this object.

E-mail address: decommer@mat.uniroma2.it.
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In this paper, we will treat the 'groupoid von Neumann algebra of the linking quantum groupoid between the dual of $SU_q(2)$ and the dual of $\widetilde{E}_q(2)$'. We also treat part of its associated infinitesimal description. One will see that it bears some similarities to the 'contraction procedure' which has been studied on the algebraic level in a series of papers by Celeghini and collaborators (see e.g. [6,7]), and in a C*-algebraic framework by Woronowicz in [47]. But our philosophy is different: while those authors consider the passage between the two quantum groups as a kind of limit procedure, we construct a concrete object linking them. In a sense, we construct a bridge to cross the water, while in the other approach, one searches a place where the river is shallow enough to cross. In any case, we will try to comment at the appropriate places when there is a concrete resemblance between our theory and the contraction procedure. We also remark that a close connection between $SU_q(2)$, $\widetilde{E}_q(2)$ and $SU_q(1,1)$ is known in relation to the q-analogue of the Askey–Wilson function transform scheme [26, Section 7].

The main tool in this paper is the theory of projective representations for locally compact quantum groups, developed in the final part of [11] (and based on observations by A. Wassermann in [42]). Indeed, using this theory, we showed in [8] (Chapter 10) that whenever a compact quantum group has an infinite-dimensional irreducible projective representation, it allows for an 'exotic deformation' into a non-compact locally compact quantum group. It turns out that there are (at least) two distinct such irreducible representations for $SU_q(2)$, which even have a quantum geometrical description: the first is associated to the homogeneous action of $SU_q(2)$ on the standard Podleś sphere [34], the other with its homogeneous action on the quantum projective plane [20]. These will be related to the deformations of $SU_q(2)$ into respectively $\widetilde{E}_q(2)$ and $\widetilde{SU}_q(1,1)$.

The contents of this paper are as follows.

In *the first section* we recall the analytic notions of von Neumann algebraic quantum groups (see [28,29,41]), von Neumann algebraic linking quantum groupoids [8] and unitary projective representations for (locally) compact quantum groups [11,8], and the algebraic notions of bi-Galois objects [36] and co-linking weak Hopf algebras [4,8]. We also recall the definitions of $SU_q(2)$ and $\widetilde{E}_q(2)$ on the Hopf $*$-algebra level, and of the dual quantized universal enveloping algebras $U_q(su(2))$ and $U_q(e(2))$.

In *the second section*, we begin by observing that the von Neumann algebraic completion of the action of $SU_q(2)$ on the standard Podleś sphere provides a projective unitary representation of $SU_q(2)$. We state the fact, proven in [10], that the standard Podleś sphere can also be seen as a subquotient $*$-algebra of a $*$-Galois object for $U_q(su(2))$, with the associated infinitesimal action coming from the Miyashita–Ulbrich action on the Galois object.

In the *third section* we combine the above two viewpoints with the general theory from [11] to construct a concrete implementation of the group von Neumann algebra of a linking quantum groupoid between $\widehat{SU_q(2)}$ and some locally compact quantum group $\mathbf{H}$.

In *the fourth section*, we show that $\mathbf{H}$ is isomorphic to the dual of Woronowicz's $\widetilde{E}_q(2)$ quantum group. We end by showing that the linking quantum groupoid is in fact cleft, i.e. that it can be implemented by a unitary 2-cocycle for $\mathscr{L}^\infty(SU_q(2))$.

Conventions and remarks on notation. For the rest of the paper, we fix a real number $0 < q < 1$. We then denote

$$\lambda = \left(q - q^{-1}\right)^{-1} < 0.$$

By ι we always mean the identity map.