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Partial hyperbolicity far from homoclinic bifurcations

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Abstract

We prove that any diffeomorphism of a compact manifold can be C^1-approximated by a diffeomorphism which exhibits a homoclinic bifurcation (a homoclinic tangency or a heterodimensional cycle) or by a diffeomorphism which is partially hyperbolic (its chain-recurrent set splits into partially hyperbolic pieces whose centre bundles have dimensions less or equal to two). We also study in a more systematic way the central models introduced in Crovisier (in press) [10].

Résumé

Hyperbolicité partielle loin des bifurcations homoclines. Nous montrons que tout difféomorphisme d'une variété compacte peut être approché en topologie C^1 par un difféomorphisme qui présente une bifurcation homocline (une tangence homocline ou un cycle hétérodimensionnel) ou bien par un difféomorphisme partiellement hyperbolique (son ensemble récurrent par chaînes se décompose en pièces partiellement hyperboliques dont les fibrés centraux sont de dimensions au plus égales à 2). Nous étudions également d'un point de vue plus systématique les modèles de dynamiques centrales introduits en Crovisier (in press) [10].

MSC: 37C05; 37C20; 37C29; 37C50; 37D25; 37D30

Keywords: Homoclinic tangency; Heterodimensional cycle; Hyperbolic diffeomorphism; Generic dynamics; Homoclinic class; Partial hyperbolicity

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0. Introduction

0.1. Towards a characterisation of non-hyperbolic systems

One of the goal of the dynamics is the study of the evolution of most systems. It appeared in the early sixties with Smale [28] that the dynamics on a compact manifold M can be well described from topological and statistical points of view for a large class of differentiable systems, the *uniformly hyperbolic* ones: these are the dynamics whose recurrence locus (the chain-recurrent set) decomposes into a finite number of invariant pieces whose tangent spaces split into two invariant subbundles $TM = E^s \oplus E^u$, the first being uniformly contracted and the second uniformly expanded. These dynamics have a nice property: they are stable (hence form an open