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Short Communication

Free and forced Rossby normal modes in a rectangular gulf of arbitrary orientation

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ABSTRACT

A free Rossby normal mode in a rectangular gulf of arbitrary orientation is constructed by considering the reflection of a Rossby mode in a channel at the head of the gulf. Therefore, it is the superposition of four Rossby waves in an otherwise unbounded ocean with the same frequency and wavenumbers perpendicular to the gulf axis whose difference is equal to $2m\pi/W$, where m is a positive integer and W the gulf's width. The lower (or higher) modes with small m (or large m) are oscillatory (evanescent) in the coordinate along the gulf; these are elucidated geometrically. However for oceanographically realistic parameter values, most of the modes are evanescent.

When the gulf is forced *at the mouth* with a single Fourier component, the response is in general an infinite sum of modes that are needed to match the value of the streamfunction at the gulf's entrance. The dominant mode of the response is the resonant one, which corresponds to forcing with a frequency ω and wavenumber normal to the gulf axis η appropriate to a gulf mode: $\eta = -\beta \sin\alpha/(2\omega) \pm M\pi/W$, where α is the angle between the gulf's axis and the eastern direction (+ve clockwise) and M the resonant's mode number. For zonal gulfs ω drops out of the resonance condition.

For the special cases $\eta = 0$ in which the free surface goes up and down at the mouth with no flow through it, or a flow with a sinusoidal profile, resonant modes can get excited for very specific frequencies (only for non-zonal gulfs in the $\eta = 0$ case). The resonant mode is around the annual frequency for a wide range of gulf orientations $\alpha \in [40^\circ, 130^\circ]$ or $\alpha \in [220^\circ, 310^\circ]$ and gulf widths between 150 and 200 km; these include the Gulf of California and the Adriatic Sea. If η is imaginary, i.e. a flow with an exponential profile, there is no resonance. In general less modes get excited if the gulf is zonally oriented.

1. Introduction

In this note we study the Rossby normal modes in a mid-latitude β-plane gulf of arbitrary orientation from a theoretical point of view. Beyond the value of advancing our knowledge in GFD, this study is also motivated by the vast literature devoted to the study of gulfs (mainly observational and numerical). Several studies (for example, Ripa, 1990, 1997) showed that the Gulf of California is forced at the mouth by an annual baroclinic Kelvin wave. Beier (1997) studied the dynamics of this gulf with a horizontal two-dimensional linear two-layer numerical model, mentioning that the β-effect has little influence

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in the dynamics and that annual long Rossby waves do not fit in the gulf (average width of 150 km) because they have a zonal wavelength of ∼650 km (calculated from the long Rossby wave dispersion relation and using a value of 30 km for the baroclinic Rossby radius); and that short Rossby waves will be subject to dissipation. Does this mean that there cannot be Rossby modes in the Gulf of California? In this paper we will show that for a gulf with the orientation and width of the Gulf of California, the $M = 2$ resonant mode is around the annual frequency for a forcing at the mouth in which the free surface just goes up and down. In a review paper Lluch-Cota et al. (2010) discussed how the Gulf of California influences the northward propagation of coastal trapped Kelvin waves associated with El Niño (ENSO) events, and how this signal results in an ENSO signature inside the Gulf. An open question in the above studies is to find out what the QG response is (given in terms of Rossby gulf modes) to this type of Kelvin wave forcing at the mouth.

As regards the Gulf of Mexico, Oey and Lee (2002) and Hamilton (2009) showed that deep eddy energy can be explained by topographic Rossby waves. Rossby wave theory has been used to study the dynamics of the Loop Current and the shedding of eddies (Hurlbutt and Thompson, 1982) in a numerical model. These are examples that show the importance of Rossby waves in the dynamics of this gulf.

There are very few references about Rossby waves in the Adriatic Sea and they refer to topographic waves. According to Pasarić et al. (2000), the propagation of topographic Rossby waves within the basin, although related to small surface displacements, could perhaps influence the adjustment at the highest frequencies they considered.

Although not strictly gulfs (they are channels), there have been some studies of Rossby waves in the South China Sea (SCS) and in the Mozambique Channel. Shu et al. (2016) observed energetic fluctuations below 1400 m from direct current measurements in the SCS that are attributable to topographic Rossby waves. Wu et al. (2008) observed free and forced Rossby Waves in the western SCS inferred from Jason-1 satellite altimetry data while Yang and Liu (2003) interpreted sea surface height anomalies in terms of forced annual Rossby waves in the northern SCS. Harlander et al. (2009) showed that a westward-propagating signal observed in the flow through the channel could be a Mozambique channel Rossby normal mode (a meridional channel mode independent of the N–S coordinate and thus with a velocity parallel to the channel only) with a period of 70 days. Like in many other papers, these are examples illustrating the importance of Rossby modes to explain observations.

Of the very few theoretical studies about Rossby normal modes in a gulf we can cite García and Graef (1998), who analysed the nonlinear self-interaction of one of such modes.

Questions like: How are the Rossby modes in a gulf?, What modes are excited when the gulf is forced at the mouth?, Are there resonant modes?, What are the frequencies and wavenumbers of these modes?, And so on are the subject of this paper. The answers to these should be of interest not only from the point of view of GFD but could also help explain the observations and definitely serve as a tool for numerical modelling efforts.

In the next section, we compute the Rossby normal modes in an idealised gulf: a parallelepiped or a rectangular box which allows analytical treatment. A Rossby gulf mode is constructed by considering the reflection of a channel mode at the gulf's head and a graphical method to find gulf modes is provided, paying careful attention to the cases when the wavenumbers parallel to the gulf's axis are complex or equal. A graph of the dispersion relation is also included. Then in Section 3 we find the Rossby gulf modes that get excited when we force the gulf at the mouth with a single Fourier component of a fixed frequency and wavenumber perpendicular to the gulf's axis. Two cases are distinguished: resonant and non-resonant forcing. We show contour maps of the resonance condition spanning a wide range of mode and physical parameters and some examples of the forced solution. Also, three special cases of forcing wavenumbers perpendicular to the gulf's axis are studied that correspond to three distinctively physical mechanisms: one in which there is no flow at the mouth, a second one in which there is a net flow through the mouth (but zero over one period) with an exponential profile resembling a Kelvin wave and a third in which there is inflow and outflow but with zero net flow at all times. We end the paper with discussion and conclusions.

2. Free Rossby modes in a gulf

Consider a β-plane with a coordinate system (x, y, z) in which x is parallel, y perpendicular to the gulf and z vertically upwards. The gulf has length L, width W, a flat bottom and it is oriented such that its axis makes an angle α with the circles of latitude (positive clockwise), see Fig. 1. To simplify matters without sacrificing the essentials, we consider a barotropic ocean with a free surface.

For the gulfs that we will be considering later for the numerical calculations of the analytical results like the Gulf of California, the Adriatic Sea or the Red Sea with their north–south extension spanning several degrees of latitude (around 10, 6 and 15, respectively), the β-plane would be more suitable than the f-plane. Besides this, we need $\beta \neq 0$, otherwise the Rossby modes will be time-independent (zero frequency).

The governing equation is the linear quasigeostrophic (QG) potential vorticity equation:

$$\partial_t \left(\nabla^2 - r_d^{-2}\right) \psi + \beta(\cos\alpha\, \partial_x + \sin\alpha\, \partial_y)\psi = 0, \tag{1}$$

where $\nabla^2 = \partial_x\partial_x + \partial_y\partial_y$, t is time, ψ the QG streamfunction, β the northward gradient of the planetary vorticity and $r_d = (gH)^{1/2}/f_0$ the barotropic Rossby radius in which g is the acceleration of gravity, H the depth of the ocean and f_0 the Coriolis parameter.