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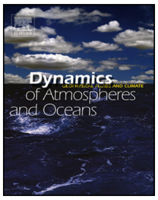

Temperature lapse rates at restricted thermodynamic equilibrium. Part II: Saturated air and further discussions

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A B S T R A C T

In the first part of this work equilibrium temperature profiles in fluid columns with ideal gas or ideal liquid were obtained by numerically minimizing the column energy at constant entropy, equivalent to maximizing column entropy at constant energy. A minimum in internal plus potential energy for an isothermal temperature profile was obtained in line with Gibbs' classical equilibrium criterion. However, a minimum in internal energy alone for adiabatic temperature profiles was also obtained. This led to a hypothesis that the adiabatic lapse rate corresponds to a restricted equilibrium state, a type of state in fact discussed already by Gibbs. In this paper similar numerical results for a fluid column with saturated air suggest that also the saturated adiabatic lapse rate corresponds to a restricted equilibrium state. The proposed hypothesis is further discussed and amended based on the previous and the present numerical results and a theoretical analysis based on Gibbs' equilibrium theory.

1. Introduction

The understanding of the processes giving the adiabatic lapse rates, as observed in the atmosphere and the ocean (Stewart, 2008; Andrews, 2010) is of fundamental importance in the physics of atmospheres and oceans. It is not yet complete in terms of thermodynamic equilibrium that has recently been discussed in literatures (Verkley and Gerkema, 2004; Akmaev, 2008; Björnbom, 2015). Related to the adiabatic lapse rate is the potential temperature, which is the temperature a fluid parcel at temperature T and pressure p would get if compressed or expanded adiabatically to a reference pressure p_r (Stewart, 2008; Holton and Hakim, 2012). In the atmosphere the saturated adiabatic lapse rate is common with its corresponding equivalent potential temperature (Salby, 1996; Andrews, 2010; Holton and Hakim, 2012). Some occasionally misunderstood theoretical issues on adiabatic lapse rates have been discussed in an interesting paper by McDougall and Feistel (2003).

A review of the history of this problem has been made by Verkley and Gerkema (2004) and this was also discussed by Björnbom (2015). J.W. Gibbs, L. Boltzmann and J.C. Maxwell and others have been involved in trying to solve this problem. Some essential points from those discussions are repeated here.

For a constant mass with a constant volume Gibbs applied the equilibrium condition of a minimum in the internal plus potential energy constrained by a constant entropy (Gibbs, 1906, p. 144). He found that such a condition is satisfied by an isothermal temperature. He also emphasized that such a condition is equivalent to a maximum in the entropy constrained by a constant mass with a constant volume and a constant internal plus potential energy.

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While Maxwell in general agreed with Gibbs' result he stated that it is by no means applicable to the case of our atmosphere (Maxwell, 1902, p. 330). He referred to the winds carrying large masses of air from one height to another. The effect of this motion in the air is that the temperature will approach a profile such that an adiabatic air parcel brought from one height to another will have the same temperature as the surrounding air. In other words the temperature profile will approach the adiabatic lapse rate. This idea Maxwell attributed to Sir William Thomson (Lord Kelvin) who called this state "Convective equilibrium of heat". However, no rigorous proof of this concept has so far been published (Verkley and Gerkema, 2004).

As described in more detail by Verkley and Gerkema (2004), in subsequent work Ball (1956) and Bohren and Albrecht (1998) introduced a constant integrated potential temperature as a constraint for the maximization of the entropy of the fluid column, again finding maximum entropy for an adiabatic lapse rate. In their recent work Verkley and Gerkema (2004) have investigated the effect of maximizing the entropy of a column of ideal gas constrained by a constant mass between constant pressure levels, a constant enthalpy and a constant integrated potential temperature.

They found an intermediate temperature profile between the adiabatic and the isothermal lapse rates as a solution of this problem. Although the isothermal lapse rate corresponds to the ultimate state of maximal entropy, processes like convective mixing prevent the atmosphere from coming close to such a thermodynamic equilibrium by lowering the maximal value the entropy can approach. This makes it natural, according to Verkley and Gerkema (2004), to include additional constraints like the constant integrated potential temperature.

This issue was studied by Akmaev (2008) in more detail on the basis of the results by Verkley and Gerkema (2004) and others. How those issues may influence the parametrization of heat transfer processes in advanced climate models is also discussed.

Björnbom (2015) treated this problem by using numerical experiments. By computer simulations he studied minima in the sum of internal plus potential energy or in internal energy alone in an isolated fluid column of given height and volume constrained by a constant mass and a constant entropy. Note that the maximization of entropy with a constant sum of internal and potential energy or a constant internal energy alone are equivalent problems. Two cases were studied: a column containing ideal gas and a column containing a liquid with constant isobaric heat capacity, constant compressibility and constant thermal expansion coefficient. While such experiments do not give mathematical rigour to the results they may contribute to paint a bigger picture and be a basis for discussing new hypotheses.

Björnbom (2015) found in both cases that the sum of internal plus potential energy of the column had a minimum for the isothermal lapse rate in agreement with Gibbs' classical criterion for thermodynamic equilibrium (Gibbs, 1906, p. 145). However, the internal energy alone also showed a local minimum in both cases for the adiabatic lapse rate, albeit with a higher value of the total column energy in the local minimum than in the isothermal case. This result led Björnbom (2015) to propose the hypothesis that the adiabatic lapse rate corresponds to a metastable thermodynamic equilibrium state restricted by passive forces represented as the slowness of heat conduction and radiative heat transfer compared to turbulent heat diffusion on the relevant time scale. In fact Gibbs already discussed such restricted equilibrium states (Gibbs, 1906, pp. 58, 142) and they have been observed in many scientific contexts, for example in chemistry (Alberty, 1989).

In the present study again the minimum in the sum of internal plus potential energy or in internal energy alone in an isolated fluid column of given height and volume constrained by a constant mass and a constant entropy has been investigated using numerical experiments. The fluid in the studied case is saturated air where the temperature of adiabatic saturated air parcels follow the saturated adiabatic lapse rate with changing height. Based on the obtained results the hypothesis on the adiabatic lapse rate as a restricted thermodynamic equilibrium phenomenon is further discussed and elaborated in order to advance the understanding of the role of thermodynamic equilibrium for the occurrence of adiabatic lapse rates.

The paper is organized with an Introduction section. After that there is an Equation section where necessary equations for the numerical study are derived followed by a Data and computational methods section. After that there is the Results section. The Discussion section is divided into five subsections. The first subsection addresses a fundamental issue on the requirements for the thermodynamic equations to be used. In the second one the proposed hypothesis from Björnbom (2015) is further discussed and amended on the basis of the results in this work.

The third subsection contains a theoretical analysis applying the equilibrium criterion for temperature by Gibbs. So-called passive forces, that are central for interpreting the results, are presented and discussed in the fourth subsection and significance of the results are addressed in the fifth one. The last section of the paper is Conclusions.

2. Equations

2.1. Equations for investigating the minima

A saturated air parcel has two degrees of freedom according to Gibbs' phase rule (Salby, 1996, Kindle Location 1286) if the components considered are air and water. Thus its specific internal energy, its specific entropy and its specific volume may be expressed as functions of the pressure and the temperature of the air parcel in analogy with the situation for a dry air parcel. As a consequence the initial value problems (14)–(16) in Björnbom (2015), being used for the calculations of the internal energy, the potential energy and the entropy of an isolated column containing ideal gas may also be used for the case with saturated air parcels.

Those initial value problems are repeated here. Consider a vertical column of fluid with a horizontal area of one m^2 and a height coordinate z. Neglecting the kinetic energy of mass elements in the column and assuming that all fluid parcels at the