Dynamics of Atmospheres and Oceans 69 (2015) 26–36

Contents lists available at ScienceDirect

Dynamics of Atmospheres and Oceans

journal homepage: www.elsevier.com/locate/dynatmoce

Temperature lapse rates at restricted thermodynamic equilibrium in the Earth system

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ARTICLE INFO

Article history:
Received 18 July 2014
Received in revised form
22 September 2014
Accepted 15 October 2014
Available online 11 November 2014

Keywords:
Adiabatic processes
Temperature gradients
Thermodynamic equilibrium
Energy
Entropy

ABSTRACT

Equilibrium temperature profiles obtained by maximizing the entropy of a column of fluid with a given height and volume under the influence of gravity are discussed by using numerical experiments. Calculations are made both for the case of an ideal gas and for a liquid with constant isobaric heat capacity, constant compressibility and constant thermal expansion coefficient representing idealized conditions corresponding to atmosphere and ocean. Calculations confirm the classical equilibrium condition by Gibbs that an isothermal temperature profile gives a maximum in entropy constrained by a constant mass and a constant sum of internal and potential energy. However, it was also found that an isentropic profile gives a maximum in entropy constrained by a constant mass and a constant internal energy of the fluid column. On the basis of this result a hypothesis is suggested that the adiabatic lapse rate represents a restricted or transitory and metastable equilibrium state, which has a maximum in entropy with lower value than the maximum in the state with an isothermal lapse rate. This transitory equilibrium state is maintained by passive forces, preventing or slowing down the transition of the system to the final or ultimate equilibrium state.

1. Introduction

The adiabatic lapse rate considered in atmosphere and ocean (Stewart, 2008; Andrews, 2010), corresponding to a constant potential temperature, is still not fully understood in terms of thermodynamical equilibrium. That has recently been discussed in the literature (Verkley and Gerkema, 2004; Akmaev, 2008). The potential temperature is simply the temperature a fluid parcel at temperature T and pressure p would get if compressed or expanded adiabatically to a reference pressure p_r (Stewart, 2008; Holton and Hakim, 2013).

Verkley and Gerkema (2004) have reviewed the history of the problem in which J.W. Gibbs, L. Boltzmann and J.C. Maxwell and others were involved. For a constant mass with a constant volume Gibbs applied the equilibrium condition of a minimum in the internal plus potential energy constrained by a constant entropy (Gibbs, 1906, p. 144). He found that such a condition is satisfied by an isothermal temperature. He also emphasized that such a condition is equivalent to a maximum in the entropy constrained by a constant mass with a constant volume and a constant internal plus potential energy.

While Maxwell in general agreed with Gibbs' result he stated that it is by no means applicable to the case of our atmosphere (Maxwell, 1902, p. 330). He referred to the winds carrying large masses of air from one height to another. The effect of this motion in the air is that the temperature will approach a profile such that an adiabatic air parcel brought from one height

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http://dx.doi.org/10.1016/j.dynatmoce.2014.10.001

to another will have the same temperature as the surrounding air. In other words the temperature profile will approach the adiabatic lapse rate. This idea Maxwell attributed to Sir William Thomson (Lord Kelvin) who called this state “Convective equilibrium of heat”. However, no rigorous proof of this concept has so far been published (Verkley and Gerkema, 2004).

The adiabatic lapse rate is observed in the atmosphere when there is much convection. An example can be seen in Fig. 13.6a in Salby’s textbook (Salby, 1996, Kindle Location 4308) which shows how the nocturnal inversion from early morning is transformed into an isentropic temperature profile during the day. Also in the ocean the adiabatic lapse rate may be found. See for example Fig. 6.9 in Stewart’s textbook (Stewart, 2008, p. 85) showing a constant potential temperature at the depths of 5000–8500 m in the Kermadec Trench in the Pacific. It is interesting that the adiabatic lapse rate has formed in such a deep trench where the intensity of convection should be on a low level. However, unlike in the atmosphere there is no considerable radiative heat transfer in the trench that may drive the lapse rate into another shape.

As described in more detail by Verkley and Gerkema (2004), in subsequent work Ball (1956) and Bohren and Albrecht (1998) introduced a constant integrated potential temperature as a constraint for the maximization of the entropy of the fluid column, again finding maximum entropy for an adiabatic lapse rate. In their recent work Verkley and Gerkema (2004) have investigated the effect of maximizing the entropy of a column of ideal gas constrained by a constant mass between constant pressure levels, a constant enthalpy and a constant integrated potential temperature.

They found that the solution of this problem is an intermediate temperature profile between the adiabatic and the isothermal lapse rates. They note that the isothermal lapse rate corresponds to the ultimate state of maximal entropy, which is the criterion for the classical thermodynamical equilibrium. However, they write, processes like convective mixing prevent the atmosphere from coming close to thermodynamic equilibrium by lowering the maximal value the entropy can approach. This makes it natural, they explain, to include additional constraints like the constant integrated potential temperature.

Akmaev (2008) has studied this issue in more detail based on the results of Verkley and Gerkema (2004) and previous work. He also discusses how those issues may influence the parametrization of heat transfer processes in advanced climate models.

The initial idea behind the present study was unambitious and simply, by numerical experiments, to confirm the classical equilibrium condition by Gibbs of a minimum in internal plus potential energy for an isolated fluid column with a constant mass and constant entropy. The equilibrium condition is an isothermal lapse rate (Gibbs, 1906, p. 145). However, the numerical experiments gave unexpected results that may shed some light on the adiabatic lapse rate in terms of thermodynamic equilibrium thus justifying dissemination.

In the present study the minimum in the sum of internal plus potential energy or in internal energy alone in an isolated fluid column of given height and volume constrained by a constant mass and a constant entropy has been investigated using numerical experiments. Note that the maximization of entropy with a constant sum of internal and potential energy or a constant internal energy alone is an equivalent problem. Two cases were studied: a column containing ideal gas and a column containing a liquid with constant isobaric heat capacity, constant compressibility and constant thermal expansion coefficient. While such experiments do not give mathematical rigour to the results they may contribute to paint a bigger picture and be a basis for discussing new hypotheses. Based on the results a hypothesis is stated on the adiabatic lapse rate as a thermodynamic equilibrium phenomenon.

The paper is organized with an Introduction section. After that there is an Equation section where necessary equations for the numerical study are derived followed by a Data and computational methods section. After that there is the Results section. The Discussion section is divided into seven subsections. In the first subsection the proposed hypothesis is presented, in the second subsection so-called passive forces, that are central for interpreting the results, are presented and discussed, in the following two subsections two examples of systems with equilibrium states that may be considered analogous to the studied system are discussed, in the fifth subsection the passive forces special for the studied systems are discussed. In the sixth subsection the significance of the results is discussed and in the seventh subsection some further important issues are discussed, for example what are the physical processes that may explain the results. The last section of the paper is Conclusions.

2. Equations

2.1. Equations for the lapse rate

An expression for the adiabatic lapse rate, $\Gamma_a = -(\partial T/\partial z)_s$ may be derived by using the following equations and noting that $ds = 0$ for a reversible adiabatic process:

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \tag{1}$$

The first partial derivative in Eq. (1) is obtained by combining the following equations for constant pressure $dp = 0$:

$$dh = c_p\, dT + \left(\frac{\partial h}{\partial p}\right)_T dp = c_p\, dT \tag{2}$$