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Convective kinetic energy equation under the mass-flux subgrid-scale parameterization

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ABSTRACT

The present paper originally derives the convective kinetic energy equation under mass-flux subgrid-scale parameterization in a formal manner based on the segmentally-constant approximation (SCA). Though this equation is long since presented by Arakawa and Schubert (1974), a formal derivation is not known in the literature. The derivation of this formulation is of increasing interests in recent years due to the fact that it can explain basic aspects of the convective dynamics such as discharge–recharge and transition from shallow to deep convection.

The derivation is presented in two manners: (i) for the case that only the vertical component of the velocity is considered and (ii) the case that both the horizontal and vertical components are considered. The equation reduces to the same form as originally presented by Arakwa and Schubert in both cases, but with the energy dissipation term defined differently. In both cases, nevertheless, the energy "dissipation" (loss) term consists of the three principal contributions: (i) entrainment–detrainment, (ii) outflow from top of convection, and (iii) pressure effects. Additionally, inflow from the bottom of convection contributing to a growth of convection is also *formally* counted as a part of the dissipation term. The eddy dissipation is also included for a completeness.

The order-of-magnitude analysis shows that the convective kinetic energy "dissipation" is dominated by the pressure effects, and it may be approximately described by Rayleigh damping with a constant time scale of the order of 10^2–10^3 s. The conclusion is also supported by a supplementary analysis of a cloud-resolving model (CRM) simulation. The Appendix discusses how the loss term ("dissipation") of the convective kinetic energy is qualitatively different from the conventional eddy-dissipation process found in turbulent flows.

1. Introduction

In examining the convective-scale processes, their energy cycle becomes a natural question. In developing a convection parameterization, for example, under the mass-flux formulation (*cf.*, Yano, 2014a), the energy cycle becomes a useful concept in order to consider a closure problem (*cf.*, Yano and Plant, 2012a; Yano et al., 2013). Such a convective energy cycle is presented for a spectrum mass-flux system by Eqs. (132) and (142) of Arakawa and Schubert (1974). Eq. (142) for the cloud work function is derived in their Appendix B. On the other hand, the kinetic-energy equation (132) is presented by Arakawa

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and Schubert (1974) without derivation. Its derivation is never formally presented in the literature in spite of the fact that this equation is quoted in the literature for many times (*e.g.*, Lord and Arakawa, 1980; Randall and Pan, 1993; Xu, 1993; Pan and Randall, 1998; Wagner and Graf, 2010; Plant and Yano, 2011, 2013; Yano and Plant, 2012b,c).

The present study is particularly motivated by our recent studies, which show that the energy-cycle system can explain the basic processes: the life cycle of an ensemble convective system consisting of discharge and recharge (*or* trigger and suppression) can be explained under a single mode truncation (Yano and Plant, 2012b), and its extension into a two-mode system explains transform from shallow to deep convection in a succinct manner (Yano and Plant, 2012c; Plant and Yano, 2013). Moreover, the convective kinetic-energy budget constitutes an important starting point for formulating their convective quasi-equilibrium hypothesis in Arakawa and Schubert (1974: see also Yano and Plant, 2012a). General importance of examining the basic formulation structure of Arakawa and Schubert's mass-flux formulation (*e.g.*, Yano, 1999, 2014a) is hardly overemphasized, with some confusions on the basic understanding (*cf.*, Adams and Rennó, 2003a; Adams and Rennó, 2003b; Yano, 2003a).

In deriving this convective kinetic-energy equation, it is important to recognize that this equation is presented in the context of discussing the closure of mass-flux convection parameterization presented by Arakawa and Schubert themselves. Though the original article may appear to suggest otherwise, these rather suggestive statements found in the paragraph associated with Eq. (132) should be best interpreted rhetorically for the consistency with the whole mass-flux formulation structure (*cf.*, Yano, 2014a).

Especially though many researchers may prefer to take the convective kinetic energy introduced here as including both vertical and horizontal components, there is no strong reason to assume so. Importantly, Arakawa and Schubert (1974) do not make any explicit remark on this, though Lord and Arakawa (1980), and Xu (1993) *do* suggest that cloud-scale kinetic energy include the horizontal component. At the same time, Lord and Arakawa (1980) emphasize "the primary source of kinetic energy generation for cumulus convection is the buoyancy force even though interaction of cumulus clouds with the vertical wind shear can be important in some situations". Yano (2014a) further emphasizes that the main focus of the mass-flux convection parameterization is the vertical motions represented by the mass flux. Thus, the present author prefers to interpret it as representing the vertical component only. At least one should be open with these two possible interpretations (*cf.*, a discussion in Section 2.2 of Yano and Plant, 2012b). The present paper is going to show that both interpretations are consistent with their Eq. (132).

Yano (2014a) shows that Arakawa and Schubert's mass-flux formulation, whose basic idea is inherited by many of the subsequent convection parameterizations, can be systematically re-derived by imposing a geometrical constraint called segmentally-constant approximation (SCA) to the cloud-resolving model (CRM). Thus, the present work also consists of applying SCA to the CRM by extending Yano (2014a).

Though such an approach is inherently limited from a point of view of a general question of the convective energy cycle, I believe it the most legitimate approach for deriving Arakawa and Schubert's Eq. (132) in the context of their paper. For this reason, for example, the production of the convective kinetic energy by background shear flow (shear production) is not explicitly included in the following analysis, although the term is likely to be important in considering, for example, tropical squall-line convection.

The kinetic-energy equation presented in Arakawa and Schubert (1974) as Eq. (132) is:

$$\frac{d}{dt}K_i = A_i M_{Bi} - \mathcal{D}_i \tag{1.1}$$

with the index i stands for a ith convective-plume element of a spectrum. Here, K_i is the convective kinetic energy, A_i is the cloud work function, as defined by Eq. (133) of Arakawa and Schubert (1974), M_{Bi} is the mass flux at the given convective base, and $\mathcal{D}_i$ is the "dissipation" rate (energy loss) of the convective kinetic energy. For a physical interpretation of the cloud work function, see Yano et al. (2005a). The types of convective plumes may be considered to include the downdrafts as well as the updrafts, in general, although Arakawa and Schubert have only considered the updrafts.

Most importantly, the "dissipation" term, $\mathcal{D}_i$, here includes all the other contributions other than the buoyant energy production given by the first term. For this reason, a quotation is always added to "dissipation" throughout the paper in order to keep in mind that a part of this term may not be dissipation at all in any physical sense. Especially, this term is far more general than what is typically considered in turbulent studies. This "dissipation" term can even be negative (*i.e.*, energy gain) in certain situations as going to be seen below.

In Arakawa and Schubert (1974), this equation is coupled with a prognostic equation for the cloud work function given by their Eq. (142):

$$\frac{d}{dt}A_i = F_i - \sum_j \mathcal{K}_{ij} M_{Bj}, \tag{1.2}$$

where F_i is a generation rate of the ith type cloud work function due to the large-scale processes, and the coefficients, $\mathcal{K}_{ij}$, represents a consumption rate of the ith type cloud work function by the jth convective-plume type. We refer to Fig. 2 of Yano and Plant (2012a) for a schematic interpretation of Eq. (1.2).

The purpose of the present paper is to present a careful derivation of the convective kinetic-energy Eq. (1.1) under the mass-flux subgrid-scale parameterization formulation (*cf.*, Yano, 2014a). The formulation is presented for the cases both