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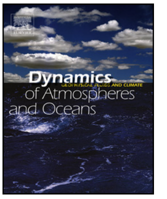

Rapid distortion theory for mixing efficiency of a flow stratified by one or two scalars

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ABSTRACT

The mixing efficiency of unsheared homogeneous turbulence in flows stratified by one or two active scalars was calculated with rapid distortion theory (RDT). For the case with one scalar the mixing efficiency η depends on the Schmidt number $Sc = \nu/D$ and the Grashof number $Gr = NL^2/\nu$, where ν is the kinematic viscosity, D is the molecular diffusivity, N is the buoyancy frequency, and L is a length scale representative of the large eddies. For the case with two scalars the efficiency also depends on the density ratio R_ρ, which compares the density difference caused by temperature and the density difference caused by salt. In the one scalar case when Gr is large, η decreases as Sc increases. The mixing efficiency increases with Gr up to a maximum value, as in numerical simulations and experiments. The maximum mixing efficiency of approximately 30% for low Sc is consistent with simulations, while the maximum efficiency of 6% for heated water is consistent with laboratory measurements. However, RDT underpredicts the maximum efficiency for saltwater and also the value of Gr at which the efficiency becomes constant. The predicted behavior of the mixing efficiency for two active scalars is similar to that for one scalar, and the efficiency decreases as R_ρ decreases, as in experiments and semi-empirical models. These calculations show that results from simulations with low Sc likely overestimate the efficiency of turbulence in strongly stratified flows in lakes and oceans.

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1. Introduction

Understanding the transport of scalars such as heat, salt, nutrients, and pollutants in environmental flows is important for predicting climate, water quality, and the health of aquatic life. Because fluxes are difficult to measure directly, a common approach is to use a mixing efficiency to estimate the eddy diffusivity and obtain vertical fluxes (Osborn, 1980). Often a constant efficiency is assumed for measurements in the ocean (e.g., Ferrari and Polzin, 2005) and lakes (e.g., Ravens et al., 2000). However, mixing and its efficiency depend on factors such as the strength of stratification, molecular diffusivity of the scalar, and the process generating the turbulence (Turner, 1973, Chapters 9–10), and questions remain about the magnitude of the mixing efficiency and its behavior in strong stratification. We use rapid distortion theory (RDT) to explore the behavior of the mixing efficiency in flows with strong stratification caused by either a single scalar or two stably stratified scalars.

Several quantities called mixing efficiency are used to study stratified flows, but their definitions vary. In devising a method to estimate the eddy diffusivity from measurements of turbulence microstructure, Osborn (1980) defined a flux Richardson number R_f as the vertical buoyancy flux $(g/\rho_0)\overline{\rho' u_3'}$ divided by the production of turbulent kinetic energy (TKE), where g is the acceleration of gravity, ρ_0 is a reference density, ρ' is the fluctuating density, u_3' is the fluctuating velocity in the vertical (or x_3) direction, and the overbar denotes an average. Ivey and Imberger (1991) generalized this definition of mixing efficiency by comparing the buoyancy flux to the sum of terms in the TKE equation other than buoyancy flux and ε, the rate of dissipation of TKE. Because the flux Richardson number measures the relative importance of terms in the TKE balance, it can vary widely during the evolution of a single turbulent event. For example, during restratification, which is a key feature of decaying turbulence in a stratified flow (Lienhard and Van Atta, 1990), it is negative.

Another definition of mixing efficiency depends on the change in mean potential energy ΔPE during a turbulent event. This change is a key quantity of interest to oceanographers (Gregg, 1987) because it measures the net effect of downgradient and upgradient fluxes on the background density profile. In experiments with a grid towed through a linearly stratified fluid, mixing efficiency has been defined as ΔPE, which is computed from density profiles measured before the tow and after the turbulence decays, divided by the work done to create the turbulence (Rehmann and Koseff, 2004). A similar definition can be applied to numerical simulations of homogeneous turbulence even though the background density gradient does not change: Stretch et al. (2010) neglected fluxes from molecular diffusion along the background gradient and computed ΔPE by integrating the buoyancy flux over the life of the turbulence; their definition of the mixing efficiency η can be written as ΔPE (per unit mass) divided by the initial TKE $q_0^2/2$:

$$\eta = \frac{\int_0^\infty (g/\rho_0)\overline{\rho' u_3'}\,\mathrm{d}t}{q_0^2/2}, \tag{1}$$

where t is time. We use this definition to characterize the net irreversible mixing from a turbulent event, and we use the symbol η to distinguish it from the flux Richardson number based on terms in the TKE equation.

Laboratory measurements of mixing efficiency and predictions from numerical simulations have similar qualitative behavior, but the maximum efficiencies differ. In experiments with towed grids the effect of stratification is quantified by a Richardson number Ri formed with length and velocity scales of the grid and the buoyancy frequency $N = [-(g/\rho_0)d\bar{\rho}/dx_3]^{1/2}$ where $\bar{\rho}$ is the background density, while the effect of molecular diffusivity D of the stratifying agent is quantified by a Prandtl number or Schmidt number $Sc = \nu/D$, where ν is the kinematic viscosity. Mixing efficiencies are small at low Ri and rise to a peak of about 6% for grid turbulence in salt-stratified fluids and temperature-stratified fluids, which have $Sc_S = 700$ and $Sc_T = 7$, respectively (Britter, 1985; Rottman and Britter, 1986; Barrett and Van Atta, 1991; Rehmann and Koseff, 2004). Simulations of Stretch et al. (2010) for $Sc = 0.5$ follow a similar trend for low Ri but reach a peak efficiency of about 5 times larger than observed in the experiments. Stretch et al. (2010) proposed that accounting for the energy used to generate surface and internal waves in the experiments would increase the efficiency and reduce the difference. Also, η decreased with increasing Sc in their full simulations with $Sc \leq 2$ and simulations with higher Sc that