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Short communication

Power-law for gravity currents on slopes in the deceleration phase

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ABSTRACT

The power-law for gravity currents on slopes is essentially an asymptotic form of the solution of thermal theory developed in Beghin, Hopfinger, and Britter (*J. Fluid Mech.* 107 (1981) 407–422), when the gravity current is sufficiently far into the deceleration phase. The power-law not only describes the long-term front location versus time relationship but also provides a useful means to estimate the buoyancy contained in the gravity current head. However, the hypothesis that gravity current is sufficiently far into the deceleration phase is hardly satisfied in experiments. In this paper, we re-formulated the power-law, considering the influence of bottom friction, and supplement the formulation by proposing a correct version of the power-law. When the gravity current is not sufficiently far into the deceleration phase, we showed that the power-law still robustly describes the front location versus time relationship, but the amount of heavy fluid in the head can be easily underestimated. The underestimation of heavy fluid in the head also depends on where the gravity current is in the deceleration phase. Therefore, a correction factor is suggested according to the location of gravity current. The amount of heavy fluid in the head, when estimated by the power-law, should be understood as the 'effective' buoyancy in driving the gravitational convection and is deemed as a lower limit for the estimation of buoyancy contained in the head.

1. Introduction

Gravity currents, also known as density currents, are gravitationally driven flows due to a density difference. A number of factors that are likely to cause variations in the density of fluid include

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temperature differentials, dissolved and suspended materials, such as salt and suspended sediments. Lock-exchange flows, in which gravity currents are produced from an instantaneous, finite buoyancy source and propagate on a horizontal boundary, have drawn much attention in the literature (see, for example, Shin et al., 2004; Marino et al., 2005; Cantero et al., 2007). Gravity currents on a slope have been considered less, but are also commonly encountered, such as powder-snow avalanches (Hopfinger, 1983) and spillage of hazardous materials (Fannelop, 1994). On a point of terminology, gravity currents on a slope is more precisely described as a 'thermal cloud', 'gravity cloud', or 'boluse' because the flow is more like a cloud with some tail following. In the literature, the terms are used interchangeably and the general term gravity current is adopted here to be consistent with the recently published work by Maxworthy (2010). For more details about the diversity of gravity currents in geophysical environments and engineering applications, the readers are referred to Allen (1985), Fannelop (1994), and Simpson (1997).

Perhaps the best-known publication on the gravity currents produced from instantaneous buoyancy sources propagating on slopes is due to Beghin et al. (1981). When the buoyancy closed in the lock is instantaneously released on a slope, the produced gravity currents go through an acceleration phase followed by a deceleration phase, according to the front velocity history. Thermal theory, which was developed therein following the famous Morton et al. (1956), has formed the basis for related gravity current studies (see, for example, Dade et al., 1994; Rastello and Hopfinger, 2004). Recently, a series of experiments aiming at gravity currents in the deceleration phase was reported in Maxworthy (2010) and the power-law which describes the front location versus time relationship and gives an estimate for the amount of heavy fluid contained in the head in the deceleration phase was proposed therein. As will be shown in the following section, the power-law for gravity currents in the deceleration phase is essentially an asymptotic form of the solution of thermal theory when the gravity current is sufficiently far into the deceleration phase, while in fact, this hypothesis is hardly satisfied in experiments. Nonetheless, it was reported that the power-law is robust (Maxworthy, 2010) even when this hypothesis is not satisfied, which begs the following questions: how could the power-law possibly be robust when the gravity current is not sufficiently far into the deceleration phase and, more importantly, when applied in this situation, does the power-law provide an accurate estimate for the amount of heavy fluid contained in the gravity current head?

This paper provides the answers to these questions and is organized as follows. The detailed derivation of power-law, including the influence of bottom friction, is presented in Section 2. The expression for an important model constant K_B, which helps determine the amount of heavy fluid contained in the gravity current head, was incorrect in Maxworthy (2010) and is corrected here. The power-law method used to estimate the amount of heavy fluid in the head is introduced in Section 3 and a correction factor is suggested when the gravity current is not sufficiently far into the deceleration phase. Conclusions are drawn in Section 4.

2. Derivation of the power-law

The configuration of the problem of a gravity current propagating on a slope is sketched in Fig. 1. Here the nomenclature mainly follows Beghin et al. (1981) for the reader's convenience. The density of ambient fluid is taken as ρ_0 and the density of heavy fluid in the lock region is ρ_1, where $\epsilon = \Delta\rho_1/\rho_0 = (\rho_1 - \rho_0)/\rho_0$. The cross-sectional area of the lock, which equivalently represents the amount of heavy fluid in the lock, is $A_0 = h_0 \times l_0$. After an instantaneous removal of the lock gate, the gravity current front develops and the gravity current head approximately takes a semi-elliptical shape with height to length aspect ratio $k = H/L$.

The convection of gravity current is driven by the heavy fluid that is contained within the head. Therefore, the linear momentum with bottom friction term takes the form

$$\frac{d(\rho + k_v\rho_0)S_1 HLU}{dt} = B\sin\theta - C_f\rho U^2 L, \qquad (1)$$

where ρ is the density of mixed fluid in the head, U is the mass-center velocity of the head, t is the time, $k_v = 2k$ is the added mass coefficient (Batchelor, 1967), $S_1 = \pi/4$ is a shape factor (Beghin et al., 1981) by which the cross-sectional area of the semi-elliptical head is defined as $S_1 HL$, C_f is the friction