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Elasto-plastic behaviour of frozen soil subjected to long-term low-level repeated loading, Part II: Constitutive modelling

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ABSTRACT

Based on the conclusions from the experimental investigation in Part I, empirical equations for the accumulated shear strain, accumulated direction ratio and elastic modulus of frozen soil are derived. The basic issues observed in the test results can also be observed in these empirical equations. Subsequently, an elaborate constitutive model for frozen soil subjected to long-term low-level repeated loading is produced by combining the empirical equations and classical elasto-plastic theory. This model accounts for the dependency of the accumulated behaviour on the initial stress state, repeated stress amplitude and frozen soil strength. In addition, the evolution behaviour of the elastic modulus with accumulated strain is also considered. The model is verified for frozen soil with the aid of the triaxial test results represented in Part I. This study is the first attempt to model the elasto-plastic behaviour under long-term low-level repeated loading for frozen soil.

1. Introduction

In Part I, the elasto-plastic behaviour, including the accumulated amount, direction and elastic modulus, of frozen soil subjected to long-term low-level repeated loading was presented based on experimental investigations. For the further development of deformation analyses and design methods for frozen soil foundations, an efficient numerical approach that can capture the long-term accumulated and elastic behaviour of frozen soil under a large number of loading cycles is indispensable. Thus, the purpose of the present study is to develop a constitutive model by combining the framework of time-dependent elasto-plastic theory and long-term mechanical behaviours of frozen soil. Although many elasto-plastic models or creep models on frozen soils under static loading have been proposed (Lai et al., 2009, 2014; Li et al., 2011; Wang et al., 2014; Yang et al., 2010), comparatively little research is done in the field of cyclic elasto-plastic models of frozen soils. Accordingly, in this subject not much experiences is available on the aspect regarding constitutive modelling of frozen soil under long-term repeated loading. Moreover, the studies on modelling the long-term elasto-plastic behaviour of the unfrozen sands or clays are also not investigated sufficiently.

Previously, a number of elasto-plastic models, including multi-surface types and bounding surface types (Dafalias, 1986; Mroz et al., 1979; Prevost, 1985; Yang et al., 2003), have been developed for sand or clay with kinematic hardening. These models can predict the details of stress–strain hysteresis loops, which require hundreds of load increments per cycle. Therefore, they have generally been used in the prediction of soil behaviour under earthquake vibrations. Compared with earthquake vibrations, these long-term soil dynamic problems represent a large number of loading cycles. These models are of limited use in problems induced by long-term vibration because of the high computing cost and uncontrollable cumulative calculation errors.

Another efficient and economic countermeasure against long computing times and error accumulation is the utilization of empirical approaches. The power equation first proposed by Monismith (1975) is frequently used to predict the permanent deformation of soil under repeated or cyclic loading. Li and Selig (1996) introduced the ratio of the cyclic deviator stress over the static deviator stress at failure into the constants of the power equation. In this method, the level of cyclic loading and physical soil state are considered. Chai and Miura (2002) further considered the effect of the initial static deviator stress and shear strength to develop a new power equation for calculating the cumulative axial strain of soft cohesive soil under repeated loading. Various types of logarithmic formulations are used to describe the evolution of strain accumulation in relation to the number of loading cycles. The parameters in most of these empirical models depend on the stress state (static and dynamic stress state), and soil properties were determined by additional experimental results (Behzadi and Yandell, 1996; Karg et al., 2010; Li et al., 2013; Niemunis et al., 2005; Sawicki and Swidzinski, 1989; Sweere, 1990). Certain researchers

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recommend the use of accumulated strain at a certain cycle ($\varepsilon^{acc}(N = 1)$, $\varepsilon^{acc}(N = 100)$ or $\varepsilon^{acc}(N = 1000)$) as a reference to predict the long-term accumulated strain, with the effects of stress state or soil properties considered directly or indirectly (Guo et al., 2013). Approaches formulated in other structures have also been proposed (Drabkin et al., 1996; François et al., 2010; Lekarp and Dawson, 1998; Paute et al., 1996). These empirical equations, however, are not frequently used, and their details are not represented in this study.

Most of the above-mentioned empirical models were proposed for accumulated axial strain, and cyclic or repeated stress was assumed to be constant during the long-term loading process. However, for the boundary-value problem, the cyclic or repeated stress distribution must be determined by a general constitutive model (elastic or elasto-plastic model) before the settlement or residue deformation prediction. This method is referred to as the “mechanistic–empirical method” and has been frequently used in previous studies (Puppala et al., 1999). However, stress redistribution induced by the accumulated strain is inevitable during long-term loading processes, and the assumption that the cyclic or repeated stress is constant is irresponsible. Therefore, an advanced model suitable for implementation in a finite element framework must be developed, and the deviatoric and volumetric portions of the accumulated strain must be considered. Marr and Chrisitian (1981), Bouckovalas et al. (1984) and Kaggwa et al. (1991) first proposed separate empirical equations for the accumulated deviatoric and volumetric strain in relation to the number of loading cycles, which are then coupled by the accumulated direction observed in recent studies. Wichtmann et al. (2006, 2014) found that the accumulated direction of sand was consistent with the flow rule of the modified Cam Clay model, which was integrated by the average stress ratio. The mechanism of accumulation for granular soil was decomposed into frictional sliding and volumetric compaction by François et al. (2010) and Karg et al. (2010), and these phenomenological laws were advanced for the volumetric portion and deviatoric portion, respectively. These authors suggested that a portion of the accumulated volumetric strain resulted from dilation induced by the deviatoric deformation, and the accumulated direction of frictional sliding is derived from the Drucker–Prager or Mohr–Coulomb criterion. Furthermore, by combining the coupled formulations for accumulated strain with a classical plasticity framework, Niemunis et al. (2005), François et al. (2010), and Karg et al. (2010) proposed accumulation models.

In previous studies, a large number of models for the resilient modulus derived from the axial stress–strain relationship were proposed to evaluate the repetitive behaviour of subgrade soils. Certain significant influencing factors, such as the stress state (static and repeated) and soil physical properties, were integrated into these models (Drumm et al., 1997; Fall et al., 2008; Guo et al., 2013; Hicks and Monismith, 1971). Poisson's ratio is another elastic property index for material, and it was assumed to be constant in the above models. However, studies (Brown and Hyde, 1975; Hicks and Monismith, 1971; Kolisoja, 1997; Sweere, 1990) have shown that Poisson's ratio is not a constant and varies with applied stress. Thus, separate mathematical formulations for defining the Poisson's ratio were proposed by certain researchers (Boyce, 1980; Hicks and Monismith, 1971). The resilient modulus and Poisson's ratio can also be replaced by the bulk and shear moduli. Boyce (1976) and Sweere (1990) developed generalized formulas for the bulk and shear modulus to define their dependence on the stress state. Nevertheless, the evolution of the elastic modulus represented in certain experimental results during cyclic or repeated loading is not considered in the above models. In the accumulation models proposed by Niemunis et al. (2005), François et al. (2010), and Karg et al. (2010), elastic behaviour is defined by the pressure-dependent stiffness and a constant Poisson's ratio, but the evolution behaviour of the elastic modulus during long-term loading processes, which significantly contributes to the stress redistribution, was not considered.

Moreover, most of the current research focuses on unfrozen soil, and a systematic attempt to model the long-term dynamic behaviour of frozen soil has not been performed. Distinct differences have been found in the long-term elasto-plastic behaviour between frozen and unfrozen soil, which are presented in Part I. Therefore, it is necessary to acquire additional information on long-term elasto-plastic behaviour and develop an efficient and economic constitutive model for frozen soil. This paper is organized as follows. First, empirical equations are proposed for accumulated shear strain, accumulated direction ratio and elastic modulus based on the experimental data in Part I. Based on these empirical equations, an efficient strategy is presented that incorporates a constitutive model for simulating the elasto-plastic behaviour of frozen soil under long-term low-level repeated loading. Subsequently, the numerical integration of this constitutive model is conducted by employing a fully explicit Euler-forward algorithm, and a reasonable integration step is determined. In the final section, the proposed constitutive model and model parameters are verified against the test results presented in Part I. In addition, the simulation capacity and validity and shortcomings of this constitutive model are discussed.

2. Empirical equations

2.1. Empirical equation for accumulated shear strain

In Part I, the accumulated shear strain increased with the number of repeated loading cycles and was significantly influenced by the initial stress ratio, initial mean stress, repeated stress amplitude and strength of the frozen soil were detected As a result, the above-mentioned influencing factors should be directly or indirectly integrated into the empirical equation for accumulated shear strain. The test results indicate that at a given number of loading cycles, additional plastic shear strain will accumulate for samples with a higher initial stress ratio. Thereafter, the function $g(\eta_0)$ (shown in Eq. (1)) is proposed in a simple method used to consider the influence of initial stress ratio on the magnitude of accumulated shear strain.

$$g(\eta_0) = a_1 \cdot \exp(b_1 \cdot \eta_0) \tag{1}$$

where a_1 and b_1 are material parameters, with $a_1 = 0.74$ and $b_1 = 1.22$; and $\eta_0 = 0.25$ is used as a reference initial stress ratio $\eta_{0,ref}$, for which $g(\eta_{0,ref}) = 1$ holds. Eq. (1) is fitted by the normalized data from the test results of Series 1 presented in Part I and shown in Fig. 1.

The magnitude of the accumulated shear strain is significantly dependent on the initial mean stress, and additional plastic shear strain is accumulated for specimens with a higher initial mean stress, which was demonstrated in Part I. Furthermore, Eq. (2) is formulated to

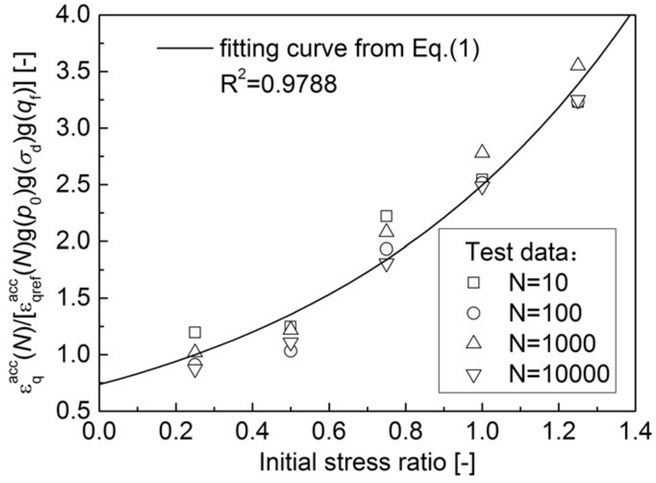

Fig. 1. Normalized accumulated shear strain versus initial stress ratio.