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Prediction of anisotropic elastic properties of snow from its microstructure

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ABSTRACT

The elastic properties of snow layers are key determinants for slab avalanche release models. This study investigates the relationships between microstructure and anisotropic elastic properties of snow. We employed micro-finite element (μFE) models built from X-ray micro-computed tomography (μCT) images to compute the effective orthotropic stiffness and compliance tensors for a wide range of snow densities and morphologies. The representativeness of the snow samples for numerical homogenization is rigorously established through the convergence analysis of the computed stiffness tensor and effective isotropic Young's modulus. The microstructure of snow is quantified in terms of ice volume fraction, ice thickness and second rank volume- and surface-based fabric tensors. The isotropic elasticity model based on ice volume fraction could explain 89% of the variability of the stiffness tensor computed by the μFE model with mean relative norm error of 43%. In contrast, the orthotropic elasticity model based on a fabric tensor and the volume fraction raised the adjusted coefficient of determination (r^2_{adj}) to 97% with mean relative norm error of 28%. Overall, the fabric based orthotropic elasticity relationship yielded better results compared to isotropic model with higher r^2_{adj}, lower relative norm errors and smaller dispersion of residuals for the prediction of stiffness tensor components as a whole as well as for the individual elastic constants. We conclude that ice volume fraction in conjunction with fabric descriptors of the snow microstructure can be used to predict the anisotropic elastic properties of snow via the relations established in this study.

1. Introduction

Snow is a material with a porous open cellular structure consisting of a complex interconnected network of sintered ice crystals. The layers in a snowpack are subjected to continual structural transformations under the influence of metamorphism and densification processes, resulting in a spectrum of snow microstructure classes (Fierz et al., 2009). The mechanical properties of snow are critical for avalanche hazard assessment (Schweizer et al., 2003) and are intrinsically linked with (a) its microstructure, which refers to the volume fractions and spatial configuration of ice and pore phases, and (b) physical properties of ice. Dry snow slab avalanches are generally released by initiation and rapid propagation of mixed-mode shear-compression fracture in a thin weak layer buried underneath a strong cohesive snow slab (McClung, 1996; Reiweger et al., 2015). The elastic properties of the slab and weak layers are key determinants for slab avalanche release models, which not only influence the transmission of deformation to the weak layer for failure initiation but are also important for fracture propagation in the weak layer (Gaume et al., 2015a,b; Habermann et al., 2008; Heierli et al., 2008; Mahajan et al., 2010; Sigrist and Schweizer, 2007).

The direct measurement of the elastic properties of weak snow classes, such as depth hoar, faceted and surface hoar crystals, from experiments is subject to large errors as sample geometry and loading conditions are often not perfect. Moreover, the pure elastic strain range for snow is very small which makes elastic loading extremely difficult to perform. The structural and mechanical properties of these snow classes also exhibit anisotropy (Reiweger and Schweizer, 2010; Srivastava et al., 2010), which plays an important role in transforming the vertical collapse deformation energy into shear deformation for crack propagation (McClung, 2005). However, physical characterization of the anisotropic stiffness (C_{ijkl}) or compliance (S_{ijkl}) tensors requires multiple measurements on the same sample that is nearly impossible because of the destructive nature of the tests. Therefore, most of the previous studies (Camponovo and Schweizer, 2001; Frolov and Fedyukin, 1998; Mellor, 1975, 1977; Scapozza and Bartelt, 2003; Sigrist, 2006) assumed snow as an isotropic material and reported quasi-static and dynamic measurements of Young's modulus of relatively well bonded snow in the vertical direction. In these studies, it was found that the Young's modulus of snow is strongly related to its density, however large unexplained variance remained which cannot be attributed solely

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to different measurement techniques. It is hypothesized that a part of scatter is caused by anisotropy of the snow samples that cannot be accounted for by a scalar quantity such as density.

An alternative is a computational approach using micro-finite element (μFE) methods, where a 3D digitized model of the microstructure is built from high resolution X-ray micro-computed tomography (μCT) images. The homogenized stiffness tensor is then computed over a representative volume element (RVE) of the microstructure for a given set of boundary conditions (Garboczi and Day, 1995). The μFE approach was first used by Schneebeli (2004) to compute the vertical Young's modulus of depth hoar snow. Recently, μFE method was applied on samples from different snow layers to calculate their effective Young's moduli and Poisson's ratios under the assumption of isotropy (Kochle and Schneebeli, 2014). Statistically reconstructed 3D snow microstructure was also used as an input geometry to compute the effective Young's modulus using mesh-free modelling (Yuan et al., 2010). However, these studies were restricted to computation of effective Young's modulus and Poisson's ratio and the evaluation of the full anisotropic stiffness tensor and its possible relation with snow microstructure was not explored.

The 3D μCT imaging allows characterization of microstructural anisotropy of porous materials by methods such as mean intercept length (MIL) (Whitehouse, 1974), star length distribution (SLD) (Smit et al., 1998) or star volume distribution (SVD) (Cruz-Orive et al., 1992). Applied to snow, these measures can describe the spatial distribution of ice and pore phases with a function that can be approximated by an ellipsoid (Harrigan and Mann, 1984) or by spherical Fourier series (Kanatani, 1984). Both approaches lead to the definition of a positive definite second rank fabric tensor that characterizes the microstructural arrangement and anisotropy in a porous solid. A preliminary study reported significant correlation between MIL fabric measures and Young's moduli of snow (Srivastava et al., 2010) and could explain the anisotropic stiffening under temperature gradient metamorphism. The granular description of structural anisotropy via contact normal tensors (Shertzer and Adams, 2011) looks very appealing, however grain segmentation and identification of grain contacts in 3D μCT images of snow microstructure is not trivial. Recently, Hagenmuller et al. (2014a) introduced a new microstructural parameter, the minimum cut density, which describes the reduced thickness of the ice matrix at bonds and showed good correlation with anisotropic Young's modulus of faceted snow. However, its relationship with all the components of the stiffness tensor is yet to be explored.

The mathematical basis for relationship between a second rank fabric tensor characterizing microstructure and the fourth rank elasticity tensor was first proposed by Cowin (1985). Following this approach, Zysset et al. (1998) developed an orthotropic elasticity model which also ensured the positive definiteness of the elasticity tensor a priori and can be reduced into (at least) a cubic symmetry model when the eigenvalues of the fabric tensor coincide. The generalized Zysset–Curnier orthotropic elasticity model (Zysset et al., 1998) consisted of five material constants besides fabric tensor and volume fraction. An extensive review by Zysset (2003) listed the formulations of existing theoretical morphology–elasticity models and compared them by applying to a common data set of trabecular bone and idealized open and closed cell 3D structures. The fabric tensor based morphology–elasticity models are very appealing as they provide an alternative to the much more computationally expensive μFE methods. In the absence of μCT-images, polar distribution of mean intercept length on 2D vertical snow sections can be used to obtain a measure of structural anisotropy. Alternatively, Kuo et al. (1998) approach could be used to approximate the MIL fabric tensor in 3D from stereological measurements on three mutually-perpendicular planar sections of snow samples.

The main objective of this study was to investigate if elastic properties of snow can be reliably predicted on the basis of either ice volume fraction alone or in conjunction with fabric tensors. We employed voxel based μFE simulations on μCT images to compute the homogenized stiffness tensors for a wide range of snow densities. The microstructural anisotropy was characterized using surface- and volume-based fabric measures. The μFE and fabric results were analysed statistically against isotropic and orthotropic morphology–elasticity relationships. Our findings confirm that ice volume fraction along with fabric are the best determinants of the anisotropic elastic properties of snow using μCT imaging.

2. Materials and methods

2.1. Snow samples

The numerical analyses were performed on a heterogeneous collection of 25 snow samples. These samples were either obtained via field sampling or prepared using controlled cold-lab experiments. A description of the samples, including their classification according to the International Classification for Seasonal Snow on the Ground (Fierz et al., 2009), is given in Table 1. The analysed samples span most of the seasonal snow classes (Fig. 1): 2 samples of precipitation particles (PP), 1 of decomposing and fragmented precipitation particles (DF), 9 of rounded grains (RG), 8 of faceted crystals (FC) and 5 of depth hoar (DH). Seven samples (HF1–HF7) were prepared from kinetic metamorphism experiments where the snow samples evolved under a fixed temperature gradient of 96 K m^{-1} (Srivastava et al., 2010). These samples correspond to various stages of transformations into facetted crystals and depth hoar. Four of the RG snow samples (ET1, T1, T2 and T3) were prepared under isothermal conditions at 264 K after sieving. Another RG sample (MTS1) was taken from the data of Chandel et al. (2014). The remaining samples comprising various snow classes were directly collected from Patsio (32 45′N, 77 16′E; 3800 m a.s.l.) and Dhundhi (32 21′N, 77 7′E; 3050 m a.s.l) field research stations in the Indian Himalayas. All the samples were scanned non-destructively with a Skyscan 1172 (Bruker, Belgium) X-ray micro-computed tomography system at resolutions ranging between 4.97 μm and 8.56 μm. The resolutions of the images were further reduced by a factor of three or four to allow reasonable computational times. The grayscale images were filtered with a 3^3 median filter and segmented into ice and pore phases. The resulting cubic volumes of side-length ranging from 5.96 mm to 9.55 mm were used for the microstructure analysis and numerical computation of elastic properties.

2.2. Microstructure parameters and construction of fabric tensors

The microstructure was characterized in terms of ice volume fraction (υ_s), ice thickness (h_{ice}), pore thickness (h_{pore}), and volume- and surface-based fabric tensors. υ_s was calculated using the hexahedral marching cube volume model (Lorensen and Cline, 1987). h_{ice} and h_{pore} defined as the mean diameter of ice structures and pores in snow respectively, were obtained using the distance transform of the ice matrix and pores (Hildebrand and Ruegsegger, 1997). The density of snow (ρ_s) was calculated by multiplying υ_s with density of ice ($\rho_{ice} = 917$ kg m^{-3}).

Fabric tensors can provide quantitative characterization of both anisotropy and orientation of the material phase of interest. In this study we used second rank MIL, SLD and SVD fabric tensors to characterize the three planes of orthotropic symmetry and degree of microstructural anisotropy. The MIL is defined as the mean distance between two solid/pore interfaces in a given direction. The distribution of the MIL at a point in 3D space forms an ellipsoid, and provides a second rank fabric tensor **H** (Harrigan and Mann, 1984). The MIL fabric tensor is defined as the inverse square root of **H**. The SLD is constructed by placing a sequence of points in the ice phase and measuring the lengths of lines emanating from the points until they encounter a solid/pore interface (Smit et al., 1998). The SVD is also constructed by placing a sequence of points in the ice phase, but instead of lines infinitesimal cones are used (Cruz-Orive et al., 1992). Because MIL traverses multiple phase boundaries, they reflect anisotropy of the configuration of the pore/solid interface,