Cold Regions Science and Technology 119 (2015) 145–150

Contents lists available at ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

Ice rubble frictional resistance by critical state theories

Sergey Kulyakhtin *, Knut V. Høyland

Sustainable Arctic Marine and Coastal Technology (SAMCoT), Centre for Research-based Innovation (CRI), Norwegian University of Science and Technology, Trondheim, Norway

ARTICLE INFO

Article history:
Received 7 April 2015
Accepted 6 August 2015
Available online 13 August 2015

Keywords:
Ice rubble
Critical state
Angle of friction

ABSTRACT

To describe the ice rubble shear strength, the Mohr–Coulomb criterion is often recommended. However, its inconsistency with some shear strength measurements was reported in the literature. The paper argues that a main source of this inconsistency is the ice rubble tendency towards volumetric changes which are ignored in the Mohr–Coulomb criterion. This argument is supported by the available data on bi-axial compression of ice rubble. The influence of volumetric changes on the ice rubble shear strength can be included by the means of the critical state concept. Several models based on this concept were reviewed; one of which, the Cam clay model, showed the best consistency with the bi-axial compression data and reported friction angles (presumably close to the critical state).

1. Introduction

Human activity in ice-covered seas sets a great challenge for the engineers designing structures for these areas. A proper design requires methods evaluating the loads from the wide variety of sea ice forms existing in nature: level ice, rafted ice, pack ice, rubble field, ice ridges etc. If icebergs and old ice ridges are not present, then first-year ice ridges give the design quasi-static load. Ice ridges consist of a sail and keel; the latter gives the major component of the ice ridge load. The keel of a first-year ice ridge consists of the upper consolidated (refrozen) layer and lower unconsolidated layer, ice rubble.

The ice rubble shear strength is the object of this study. To predict the shear strength of ice rubble ISO/FDIS/19906 (2010) recommends the Mohr–Coulomb criterion. This criterion establishes the relationship between shear (τ) and normal (σ_n) stresses which holds on the failure plane:

$$\tau = \pm(c + \sigma_n \tan\phi) \qquad (1)$$

where ϕ and c are the two material properties—friction angle and cohesion. However, the application of this criterion to ice rubble is problematic because ice rubble laboratory tests report values of friction angles from 11° to 65° and cohesions from zero to 4 kPa (e.g. review by Ettema and Urroz, 1989). ISO/FDIS/19906 (2010) attributes this variation to the ambiguity of separation between frictional and cohesional components of total shear strength. Another explanation is that majority of ice rubble tests are direct shear, and their results dependent on a particular apparatus because of non-uniform stress and strain fields (Løset and Sayed, 1993). But even limiting their analysis to unconsolidated ice rubble and a single apparatus, Timco and Cornett (1999) have shown that the friction angle of ice rubble still vary systematically with boundary conditions. This discussion leads to the conclusion that a simple criterion, such as Mohr–Coulomb, cannot accurately describe the complex process of ice rubble shear-deformation.

A number of physical phenomena contribute to the ice rubble shear strength: friction/adhesion between ice surfaces, phase changes in ice fragments and surrounding water, rearrangement and crushing of ice fragments. We do not attempt to address all mentioned phenomena, but we will show that the accounting for ice fragments rearrangement and crushing, by introducing a volumetric parameter, will improve the prediction of ice rubble shear strength.

Surprisingly, volumetric deformations are rarely measured in the laboratory studies of ice rubble. Gale et al. (1987) have reported the triaxial compression tests which include the volumetric and axial strains. However, they have used pre-confinement stresses (from 20 kPa to 800 kPa) and durations (from 2.5 to 31 h) which make the effects of creep and adhesion prominent. As those effects are ignored in our study, data by Gale et al. (1987) will not be considered here. Some measures of volumetric deformations are included in few direct shear box tests (Pustogvar et al., 2014; Wong et al., 1987; Yasunaga et al., 2002), but those measurements are scarce and, as mentioned, dependent on an apparatus used; so they will not be discussed further as well. The last type of ice rubble tests that measure volumetric deformations is performed in the bi-axial compression apparatus developed by Timco et al. (1992). The measurements from this apparatus are most suitable for our objectives and will be exploited further.

Based on the bi-axial compression data, several studies (Løset and Sayed, 1993; Timco and Cornett, 1999) argue that the Mohr–Coulomb

---

* Corresponding author at: Høgskoleringen 7A, 7491 Trondheim, Norway. Tel.: +47 735944554.
E-mail address: sergey.kulyakhtin@ntnu.no (S. Kulyakhtin).

http://dx.doi.org/10.1016/j.coldregions.2015.08.003

criterion cannot describe the ice rubble shear strength. On the other hand, Liferov and Bonnemaire (2005) by analysing data from the same apparatus argue in favour of the Mohr–Coulomb criterion. The lack of agreement between scholars and the lack of an alternative to the Mohr–Coulomb criterion for ice rubble description motivates further investigation. Therefore, we have two main objectives in this paper: a) to re-evaluate the available bi-axial compression data and show that when considerable volumetric changes occur the Mohr–Coulomb criterion cannot be applied; b) to provide an overview of some alternative theories based on the critical state concept and show that these theories can overcome difficulties which the Mohr–Coulomb criterion encounters interpreting the data of ice rubble bi-axial compression.

2. Re-evaluation and summary of bi-axial compression data

2.1. Data overview and plane strain parameters

The apparatus for bi-axial compression test of ice rubble has a 1 m × 1 m × 0.5 m chamber which is equipped with activators, displacement transducers and load cells; the full description can be found in Timco et al. (1992). This apparatus was used in two published studies: Sayed et al. (1992) and Løset and Sayed (1993). Both these studies used the same type of boundary conditions—proportional strains (the ratio between major and minor strains is kept constant for each test); however, they used different types of ice rubble.

Sayed et al. (1992) used EGAD model ice, which was grown in sheets of 30–40 mm thickness. Subsequently, these sheets were manually broken into pieces with the largest dimension varying from 50 mm to 250 mm.

Løset and Sayed (1993) used freshwater ice fragments of two types: almost cubic approximately 100 mm × 100 mm × 130 mm blocks (the largest dimension ≈ 190 mm) and smaller fragments of random shapes which had the largest dimension of about 25 mm. From the two types of ice fragments, three different configurations were made: large (≈190 mm), small (≈25 mm), and a mixture of large and small fragments.

Both of the studies tested ice rubble in dry and submerged conditions. However, Løset and Sayed (1993) do not report submerged tests because the results of submerged tests are similar to those of dry tests for the same rubble configurations. Table 1 gives an overview of the data and sources.

In all of these studies the results were reported using standard plane strain invariants: maximum shear stress (t) and mean stress (s) in the plane of shearing

$$t = \frac{\sigma_1 - \sigma_3}{2} \qquad s = \frac{\sigma_1 + \sigma_3}{2} \tag{2}$$

where σ_1 and σ_3 are major and minor stresses; volumetric strain and shear strain

$$v = \varepsilon_1 + \varepsilon_3 \qquad \gamma = \varepsilon_1 - \varepsilon_3 \tag{3}$$

where ε_1 and ε_3 are major and minor strains. They also assumed cohesionless material so that the Mohr–Coulomb criterion Eq.(1) in plane strain conditions can be written as:

$$\sin\phi_m = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{t}{s} \tag{4}$$

which can be also seen as the definition of mobilised friction angle, ϕ_m.

2.2. On initial values of ϕ_m

The dependencies of shear stress (t) on mean stress (s) in bi-axial compression tests can be well approximated by straight lines, except for the beginnings of tests (Løset and Sayed, 1993 and Sayed et al., 1992). A constant t/s ratio means that the mobilised friction angle (ϕ_m) is constant (Eq. 4). This fact has been previously noticed by Cornett and Timco (1996), and they use a “limit friction angle” term for the averaged values of ϕ_m which correspond to a constant level. During the initial stage of bi-axial compression tests, the values of ϕ_m are not constant and reach values up to 90°. Liferov and Bonnemaire (2005) argue that the high values of mobilised friction angles are the result of disregarding cohesion in Eq. (4). There could have been some initial cohesion even though these experiments were done on unconsolidated ice rubble (room and ice temperatures of −2 °C and water temperature of 0 °C) with low pre-confinement (only ice rubble weight/buoyancy). However, for us it seems more reasonable that the initial phase was the time it took to develop a uniform stress field inside the ice rubble sample. All tests showing this behaviour were performed using negative strain ratio, meaning that one wall of the chamber was compressing ice rubble sample and another was moving away from it. Therefore, considering the big sample volume, it is reasonable to assume that the initial values of minor stress (σ_3) were equal to zero. This situation could persist until an ice rubble sample has reached sufficient confinement to transmit stresses from the compressing wall. This scenario can be clearly seen in (σ_1, σ_3) plots reported by Sayed et al. (1992) and by Løset and Sayed (1993). When σ_3 is equal to zero the definition of mobilised angle of friction, Eq. (4), will exactly result in 90°. This explanation is also supported by the high initial porosities of ice rubble (almost 0.4) which are reported by Løset and Sayed (1993).

Whether the initial high values of friction angles should be attributed to cohesion or to the development of uniform stress fields, it is not important in the context of continuous deformations. The behaviour of ice rubble during continuous deformations will be the main concern in succeeding sections.

2.3. Summary of ϕ_m data and deficiency of Mohr–Coulomb criterion

Following the discussion in previous section, we want to summarise the measurements of ϕ_m excluding the initial stage. The results of Sayed et al. (1992) and of Løset and Sayed (1993) show that, for the most tests, a linear relation between shear stress and mean stress, resulting in constant ϕ_m, is reached after approximately 5% of shear strain. Therefore, we summarise values of ϕ_m for each tests by taking an average value after 5% of shear strain (Fig. 1).

The mobilised friction angles are plotted versus the ratio of the strain increments $\delta\varepsilon_3/\delta\varepsilon_1$, which is constant for each test owing to

Table 1
Overview of the data.

Source	Sayed et al. (1992)		Løset and Sayed (1993)		
Dry/wet	Dry	Wet	Dry	Dry	Dry
Type of ice	EGAD	EGAD	Fresh	Fresh	Fresh
Dimensions (mm)	Thickness largest dim.	30–40 50–250	Cubic, large 100 × 100 × 130	Random, small, largest dim. 25	Random, mixed, largest dim. 25–190