Cold Regions Science and Technology 106–107 (2014) 22–27

Contents lists available at ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

Analysis of random temperature field for freeway with wide subgrade in cold regions

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ARTICLE INFO

Article history:
Received 24 October 2013
Accepted 10 June 2014
Available online 18 June 2014

Keywords:
Stochastic finite element methods
Cold regions
Wide-subgrade of freeway
Random temperature fields

ABSTRACT

In cold regions, the freeway with wide pavement has larger heat absorption area than the normal, resulting in more difficulties to ensure the stability of embankment. However, the conventional finite element methods for stability analysis are always deterministic, rather than taking random parameters and conditions into account. In this paper, using stochastic finite element methods, the parameters of boundary conditions are considered as random variables, and the random temperature fields for wide-subgrade freeway are obtained and analyzed. The results show that heat accumulation effect is obvious in the central section of subgrade and that the closer it is to the upper boundary of the roadbed, the greater the temperature variance is. Besides, both effects increase with time.

1. Introduction

In The Plan of National Expressway Network, approved by the State Council of the People's Republic of China in 2004, there would be approximately 30% of freeway running across the seasonally frozen ground and 2% crossing permafrost areas. Many provinces and regions in China would benefit from this plan, like Northeast of China, Tibet Autonomous Region, Xinjiang Uyghur Autonomous Region, the Qilian Mt. and the Tianshan Mt. The most important characteristic of freeway in cold regions is its higher heat absorption capacity because of its wider and higher embankment, resulting in the more observable interaction between highway and permafrost and more intensive negative heat transfer. The study on the heat transfer process in embankment with different types and widths of pavement in permafrost regions indicated that when the width of pavement is doubled, the heat intensity at bottom of embankment increases by 0.6 times and the annual average heat flow in bottom of bituminous concrete pavement embankment is about 3 times to that of sand pavement (Yu et al., 2006). On the one hand, owing to the increasing of the width of pavement, the heat transfer between embankment and nearby frozen soil becomes more difficult. On the other hand, the effect of heat accumulation in embankment rapidly leads to a degradation of permafrost, which means permafrost under embankment tends to thaw and the permafrost table to decline, and will be even more prone to thawed settlement and frost heaving which therefore results in uneven roadbed settlement. Consequently, the construction of freeway with wide subgrade in cold regions faces a more complicated engineering problem than that of ordinary roads (Yu et al., 2010). So, as the thermal stability of embankment is analyzed through numerical methods, the determination of material properties and boundary conditions directly affect the stability analysis of the freeway in cold region. But nearly all of the thermal analysis of engineering in cold regions is developed under the assumption that the model's parameters, such as material properties and boundary conditions, are deterministic quantities, therefore, their results are determined. However, the property parameters of frozen soil are variable spatially and temporally, and the boundary conditions are random, and therefore, such variability should be considered in the numerical analysis of frozen embankment models. Thus, in cold regions, it is necessary to simulate the stability of freeway with wide embankment by taking account of the random variables mentioned above. In this paper, based on the earlier studies of random temperature field of ordinary embankment (Liu et al., 2006, 2007), the author uses the stochastic finite formulation derived from the perturbation technique and takes the parameters of boundary conditions as random variances, and the random temperature fields for wide-embankment freeway were obtained and analyzed.

2. The stochastic finite element equations for random temperature field

The problem considered in this paper is thus governed in the region Ω, by the non-linear parabolic equation (without considering the rate of internal heat generation):

$$C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) \tag{1}$$

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http://dx.doi.org/10.1016/j.coldregions.2014.06.004

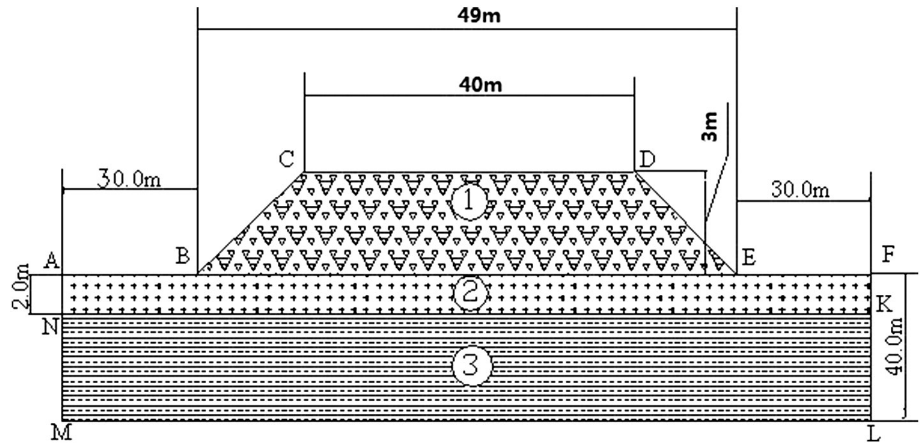

Fig. 1. The computational model.

subjected to boundary condition:

$$T = T_w \tag{2}$$

on part of boundary Γ_1,

$$-\lambda \frac{\partial T}{\partial n} = q \tag{3}$$

on part of the boundary Γ_2 and

$$-\lambda \frac{\partial T}{\partial n} = \alpha(T - T_a) \tag{4}$$

on part of boundary Γ_3, where T is the non-linear temperature field, C is volumetric heat capacity, λ is the thermal conductivity, T_w is boundary temperature, T_a is the equilibrium temperature and q represents the imposed heat flux, respectively.

Initial conditions in Ω is

$$T|t = 0 = T_0. \tag{5}$$

Using the perturbation stochastic finite element method, considering b_r ($r = 1, \cdots, R$) as random variables (such as the material specific heat capacity, thermal conductivity, and boundary condition parameter), the following stochastic finite element governing the transient heat transfer process is obtained (Liu et al., 2006).

Zeroth-order function

$$\mathbf{C}\dot{\overline{\mathbf{T}}} + \mathbf{K}\overline{\mathbf{T}} = \overline{\mathbf{Q}}. \tag{6}$$

First-order function

$$\mathbf{C}\dot{\mathbf{T}}_r' + \mathbf{K}\mathbf{T}_r' = \mathbf{Q}_r' - \left(\mathbf{C}_r'\dot{\overline{\mathbf{T}}} + \mathbf{K}_r'\overline{\mathbf{T}}\right) \quad r = 1, 2, \cdots, R \tag{7}$$

where

$$C_{ij} = \int_\Omega \overline{C} N_i N_j d\Omega \tag{8}$$

$$K_{ij} = \int_\Omega \lambda \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} \right) d\Omega + \int_{\Gamma_3} \overline{\alpha} N_i N_j d\Gamma \tag{9}$$

$$\overline{Q}_i = \int_{\Gamma_3} \overline{\alpha}\,\overline{T}_a N_i d\Gamma - \int_{\Gamma_2} \overline{q} N_i d\Gamma \tag{10}$$

$$(K_r')_{ij} = \int_\Omega \lambda_r' \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} \right) d\Omega + \int_{\Gamma_3} \alpha_r' N_j N_j d\Gamma \tag{11}$$

$$(C_r')_{ij} = \int_\Omega C_r' N_i N_j d\Omega \tag{12}$$

$$(Q_r')_i = \int_{\Gamma_3} (\alpha_r'\overline{T}_a - \overline{\alpha} T_{ar}') N_i d\Gamma - \int_{\Gamma_2} q_r' N_i d\Gamma. \tag{13}$$

The symbol ($'$) represents the items on random variables' partial derivatives. Such as the following expression

$$\mathbf{T}_r' = \begin{Bmatrix} \dfrac{\partial T_1}{\partial br} \\ \vdots \\ \dfrac{\partial T_n}{\partial b_r} \end{Bmatrix} (r = 1 \quad \cdots \quad R) \tag{14}$$

which is the column vector for node temperature on random variable b_r partial derivatives.

The auto-covariance matrices for the nodal temperature are given by

$$Cov\left(T^i, T^j\right) = \sum_{k=1}^{R}\sum_{l=1}^{R} \frac{\partial T^i}{\partial b_k}\frac{\partial T^j}{\partial b_l} Cov(b_k, b_l) \tag{15}$$

Table 1
Thermal parameters of media in embankment.

Physical parameters	λ_f(W/m · °C)	C_f(J/m^3 · °C)	λ_u (W/m · °C)	C_u (J/m^3 · °C)	L (J/m^3)
Gravel soil	1.980	1.913×10^6	1.919	2.227×10^6	20.4×10^6
Sub-clay	1.351	1.879×10^6	1.125	2.357×10^6	60.3×10^6
Weathering meta-morphic schist	1.824	1.846×10^6	1.474	2.099×10^6	37.7×10^6