Cold Regions Science and Technology 106–107 (2014) 47–54

Contents lists available at ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

A simple rheological element based creep model for frozen soils

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ARTICLE INFO

Article history:
Received 16 September 2013
Accepted 22 June 2014
Available online 28 June 2014

Keywords:
Creep
Frozen soils
Constitutive model
Field loading test

ABSTRACT

Creep of frozen soils is one of the most important issues in cold regions engineering. This paper first reviews the state-of-the-art of creep models for frozen soils. It is found that the elementary rheological models are more feasible in describing the creep of frozen soils. The authors propose a simple model by combining Maxwell, Kelvin and Bingham body, with a parabolic yield criterion. The model is verified by the direct shear creep tests on frozen fine sand, and the calculated creep strains agree in general with test data. A field loading test in Beiluhe site along Qinghai–Tibet railway is modeled on FLAC platform with the proposed model and the simulated settlement of underlying warm and ice-rich permafrost well coincide with the in situ monitored data.

1. Introduction

Since the last century, many engineering problems are arising as the permafrost degenerates, e.g., settlement of permafrost foundations, frost boiling of subgrade as well as thaw slumping of slopes, of which settlement is one of the main concerns for frozen ground engineers (Cheng et al., 2008). As for the sources of settlement, many studies focused on the thaw settlement of permafrost (Morgenstern and Nixon, 1971; Nixon and Morgenstern, 1973; Yao et al., 2012). Creep of frozen soils was not generally taken into account. However, when warm frozen soils are involved, i.e., frozen soils at temperatures higher than the threshold of −1.0 °C (Qi and Zhang, 2008), creep may also contribute a big part to the total settlement (Ladanyi, 1983). Furthermore, as a common source of settlement of foundations in cold regions, creep of frozen soils should always be considered (Qi et al., 2007).

Creep models for conventional soils can roughly be categorized as one of the following types (Liingaard et al., 2004): (1) empirical models, (2) elementary rheological models, and (3) general stress–strain–time models. Empirical models are mainly obtained by fitting creep test data and closed-form models are generally given under specific boundary and loading conditions, e.g., Yin's model (Yin, 1999). Empirical models often precisely describe creep behaviors of soils but are limited in further extension. As for the elementary rheological models, they were developed for fluids, steel and metals and usually given in differential form. This kind of model can be characterized by a series of mechanical elements, e.g., Hookean spring, Newtonian dashpot and Saint Vernant's slider. The general stress–strain–time models are often in incremental form and usually implemented in numerical software, such as the overstress model proposed by Karstunen and Yin (2010) and the further extended overstress model by Yin et al. (2011).

As for frozen soils with ice inclusion, whether the aforementioned models are applicable or not depend on how thermal effect is reflected. In cold regions engineering, empirical models are mostly used by engineers. Andersland and Akili (1967) introduced the theory of rate process to frozen soil mechanics. Fish (1980, 1983) viewed the deformation of frozen soils as entropy change and proposed an exponential creep model. Miao et al. (1995) deduced a damage evolution equation by taking into account the variation of ice content. However, the microscopic variables in the models above are difficult to be obtained in laboratory. Thus, some simpler models are put forward by fitting test data. Assur (1980) put forward a tertiary creep model for frozen soils and the failure conditions are also included. Ting (1983) then proposed a model which can better describe the creep failure under relatively high stresses. Zhu and Carbee (1983) classified the creep of frozen silt as short-term and long-term creep by the turning point of the relationship between the minimum strain rate and the applied stress and found that the Assur model well describes the short-term creep but deviates for long-term creep, and then the empirical models were proposed for both cases.

Due to the limitation of empirical models in extension, some elementary rheological models were developed in describing the creep of frozen soils. Vyalov (1986) derived an elementary rheological model for frozen soils. Li et al. (2011) deduced an improved Nishihara creep model for frozen deep clay by combining a generalized Kelvin model and an improved viscoplastic body. So far, based on the hypothesis on the effect of spherical and deviatoric stresses (Lai et al., 2010),

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http://dx.doi.org/10.1016/j.coldregions.2014.06.007
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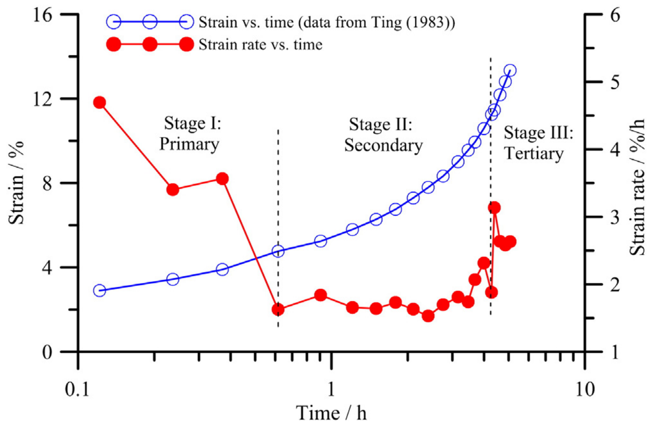

Fig. 1. Creep stages for frozen soils.

this kind of model has been extended into 3D conditions. In the case that the incremental form is given, these models can be classified as the general stress–strain–time model. Due to its clear physical meaning and numerical convenience in programming, it is recommended for engineering practice.

Therefore, in this paper, the authors attempt to build a simple rheological element based model for the creep of frozen soils. Its rationality is verified by direct shear creep tests. A field loading test is further modeled for engineering application.

2. Rheological element based creep model for frozen soils

Fig. 1 presents the creep stages for frozen soils. We can see that under a constant load, frozen soils initially deform instantaneously followed by the delayed deformation with strain rate decreasing, corresponding to stage I; when the strain rate minimizes to a constant, the viscoplastic flow occurs in stage II; afterwards, the strain rate increases rapidly and creep failure is attained, as is classified as the stage III. The above processes vary with stress levels. Fig. 2 presents the typical creep curves of frozen sand under various stress levels ($T = -10$ °C, $\sigma_3 = 1.5$ MPa). For stresses lower than the long-term strength, the tertiary stage, with increasing strain rate, may not develop, whereas frozen soils exhibit all creep stages, during which a short-term failure occurs under relatively high stresses (Vyalov, 1986).

According to the typical features of creep of frozen soils, the elementary rheological creep model is identified as follows:

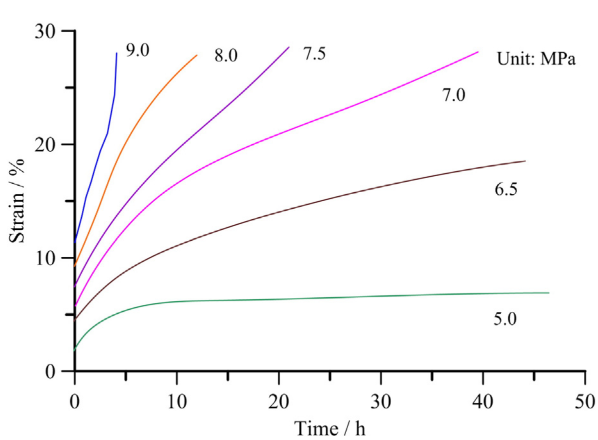

Fig. 2. Creep curves for frozen sand under triaxial conditions (Ma et al. 1994).

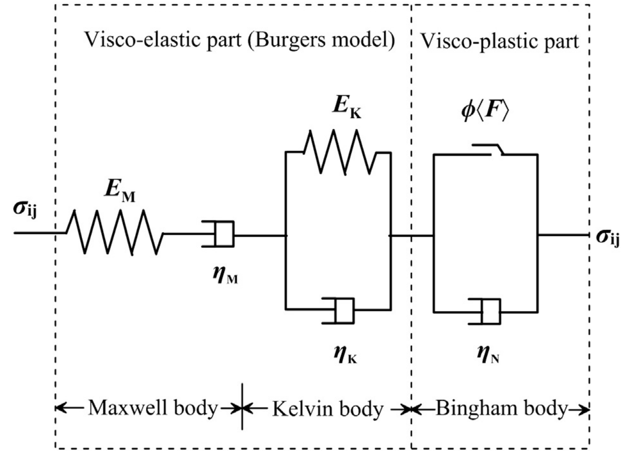

Fig. 3. Conceptual structure of the creep model for frozen soil.

1) Instantaneous deformation may include elastic and plastic components. The plastic part is only considered when the bearing failure of foundation occurs (Andersland and Ladanyi, 1994) while for the cases under conventional foundation load it is generally neglected for convenience. Here, we select the Hookean spring, the material of classical elasticity, to represent the elastic part of the instantaneous deformation in stage I.
2) Under relatively low stresses, the strain of frozen soils tends to be stable such as the creep curve of frozen sand under a deviatoric stress of 5 MPa in Fig. 2. Based on this feature, we introduce a Kelvin body which predicts a constant strain in the infinite time limit.
3) We can see from Figs. 1 and 2 that there is a relatively stable stage during creep of frozen soils (stage II), in which the strain rate is a constant for a given stress level, depending on the yield function $\phi(F)$. If $\phi(F) \le 0$, the strain rate tends to zero, whereas it increases positively with stress level (Ma et al. 1994). Thus, a dashpot is needed for the viscoplastic flow with strain rate close to zero while the Bingham body is taken for that with flow rate varying with stress conditions which exceed a yield stress level.
4) The tertiary stage for frozen soils (stage III) occurs under relatively high stresses. As for the settlement of cold regions engineering, it is generally not taken into account under conventional foundation load.

Combining all the identified elements, we obtain the conceptual structure of the simple creep model for frozen soils, as shown in Fig. 3.

In the case of $\phi(F) \le 0$, the model is actually the Burgers model while the viscoplastic strain represented by Bingham body is taken into account only when $\phi(F) > 0$. The creep model for frozen soils under triaxial conditions can be deduced as

$$e_{ij} = \frac{\sigma_{ij}}{2E_M} + \frac{\sigma_{ij}}{2\eta_M}t + \frac{\sigma_{ij}}{2E_K}\left[1 - \exp\left(-\frac{E_K}{\eta_K}t\right)\right](\phi(F) \le 0) \quad (1-a)$$

$$e_{ij} = \frac{\sigma_{ij}}{2E_M} + \frac{\sigma_{ij}}{2\eta_M}t + \frac{\sigma_{ij}}{2E_K}\left[1 - \exp\left(-\frac{E_K}{\eta_K}t\right)\right] + \frac{1}{2\eta_N}\langle\phi(F)\rangle\frac{\partial Q}{\partial\{\sigma\}}t(\phi(F) > 0) \quad (1-b)$$

in which E_M and E_K are elastic modulus for Maxwell and Kelvin bodies, respectively; η_M, η_K and η_N are the coefficients of viscosity for Maxwell, Kelvin and Bingham bodies, respectively; and Q is a viscoplastic potential function. $\phi(F)$ is the scaling function representing the magnitude of