Cold Regions Science and Technology 109 (2015) 43–52

Contents lists available at ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

Stochastic analysis model of uncertain temperature characteristics for embankment in warm permafrost regions

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ARTICLE INFO

Article history:
Received 21 March 2014
Accepted 24 September 2014
Available online 2 October 2014

Keywords:
Uncertain temperature characteristic
Random field
Local average
Embankment in warm permafrost region
Stochastic finite element

ABSTRACT

For embankments in cold regions, the soil properties and the upper boundary conditions are stochastic because of complex geological processes and changeable atmospheric environment. In this study, we model the soil properties as random fields and the upper boundary conditions as stochastic processes. A triangular local average (TLA) method is used to discretize the two-dimensional (2D) random fields. The random temperature fields of an embankment in a cold region are investigated by Neumann stochastic finite element method (NSFEM), and the computational formulas of mean and standard deviation are developed. In the calculation flow chart, a stochastic finite element (FE) program has been compiled by Matrix Laboratory (MATLAB) software. The results show that TLA method perfectly matches with triangular FE method. The randomness of soil properties and boundary conditions lead to the randomness of temperature. The results will improve our understanding of the random temperature field of embankments in cold regions. The proposed method can be used to solve other uncertain thermodynamic problems.

1. Introduction

In cold regions engineering construction, particularly Qinghai–Tibetan railway, road and tunnel in China, the temperature is important to determine the stability. Because of the complexity of the analytical solution for the temperature field, it is important to apply numerical methods, such as finite element (FE) method, to analyze the thermal stability for each engineering construction. But most of the thermal analysis of engineering in cold regions is developed under the assumption that the model's parameters, such as material properties and boundary conditions, are deterministic quantities (e.g., Lai et al., 2003, 2004; Mu, 1988; Zhang et al., 2002, 2005). In fact, the property parameters of soil are variable because of the complex geological processes (Dasaka and Zhang, 2012; Elkateb et al., 2003; Ramly et al., 2002; Soulie et al., 1990; Vanmarcke, 1983). Also, the upper boundary conditions of an embankment are stochastic due to the changeable atmospheric environment (Hasselmann, 1976; Majda et al., 1999, 2001). In warm permafrost regions, the random soil properties and boundary conditions can truly make the states of warm frozen soil become stochastic, i.e., it is sometimes frozen soil, sometimes thawed soil. Therefore, it is significant important to consider the random aspects of the parameters and conditions. But the conventional FE method cannot be used to solve these problems.

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Some researchers have tried to consider the random aspects as random variables (e.g., Hien and Kleiber, 1997; Liu et al., 2006; Madera and Sotnikov, 1996). Others have tried to consider the random aspects as stochastic processes (e.g., Emery, 2004; Xiu and Karniadakis, 2003). The air is uniform enough compared with the soil. So the upper boundary conditions of embankment can be approximately modeled as stochastic processes. But these methods cannot consider the spatial variability of soil properties. Random field method can quantify the correlation between any two observations in a field. The earliest papers on the application of random field theory to geotechnical practice appeared in the 1970s (Vanmarcke, 1977) and further research progress has been made in recent years (e.g., Griffiths and Fenton, 2004; Phoon and Kulhawy, 1999; Vanmarcke et al., 1986; Zhu and Zhang, 2013).

In cold regions, very few people have considered the property parameters of frozen soil as random fields because it is unusual and complex. Liu et al. (2007) used the random field method, but the numerical characteristics were not mentioned. In addition, the results obtained from first-order perturbation technique were not accurate when the perturbation was more than 20% to 30%. Wang and Zhou (2013) introduced a new method to calculate the random temperature field of frozen soil. But the structure was discretized into triangle unit and the random field was discretized into quadrangle unit. They are inconsistent. In order to solve this problem, a research result was applied in that paper (Liu and Liu, 1993). However, the numerical characteristic of triangle unit is inaccurate, and the calculation program is complex.

In this paper, the TLA method is presented to discretize the 2D random fields. Its main advantage is that triangular random field element

http://dx.doi.org/10.1016/j.coldregions.2014.09.013
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can match with triangular FE. Considering the soil properties as random fields and the upper boundary conditions as stochastic processes, the random temperature fields of an embankment in a cold region are obtained by NSFEM. The mean and standard deviation of the random temperature field are analyzed. The results provide a theoretical basis for engineering reliability analysis and design. The stochastic analytical method can be widely applied to other similar engineering construction questions in cold regions.

2. Deterministic governing differential equations and finite element formulae

2.1. Governing differential equations

In warm permafrost regions, the temperature fields of embankment are considered as a nonlinear problem of heat transfer with phase change. Taking into account phase change, the differential equations of two-dimensional temperature fields are given by

In Ω_f,

$$\frac{\partial}{\partial x}\left(\lambda_f \frac{\partial T_f}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_f \frac{\partial T_f}{\partial y}\right) = C_f \frac{\partial T_f}{\partial t} \tag{1}$$

In Ω_u,

$$\frac{\partial}{\partial x}\left(\lambda_u \frac{\partial T_u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_u \frac{\partial T_u}{\partial y}\right) = C_u \frac{\partial T_u}{\partial t} \tag{2}$$

where f and u represent the frozen and the unfrozen states, respectively. T_f, C_f and λ_f are the temperature, volumetric heat capacity and thermal conductivity of embankment in the frozen area Ω_f, respectively. Parameters with subscript u are the corresponding physical components in the unfrozen area Ω_u. t is time and x, y, and z are distances.

At the frost front position $s(t)$, the continuous condition and the conservation of energy should be met, i.e.,

$$T_f(s(t), t) = T_u(s(t), t) = T_m \tag{3}$$

$$\lambda_f \frac{\partial T_f}{\partial n} - \lambda_u \frac{\partial T_u}{\partial n} = L \frac{ds(t)}{dt} \tag{4}$$

where L is the latent heat per unit volume. T_m is the freezing point of soil. Its mean value is taken as -0.1 °C in this paper.

It is assumed that the phase change occurs in a range of temperatures ($T_m \pm \Delta T$) based on the method of sensible heat capacity. When the equivalent heat capacity is constructed, the effect of the interval temperature, ΔT, should be included. Suppose that C_f, C_u, λ_f, and λ_u do not depend on temperature T. Then, in the interval: $T_m - \Delta T \le T \le T_m + \Delta T$, the following definitions may be assumed (Bonacina et al., 1973).

$$C = \begin{cases} C_f & T < T_m - \Delta T \\ \dfrac{C_u + C_f}{2} + \dfrac{L}{2\Delta T} & T_m - \Delta T \le T \le T_m + \Delta T_u \\ C_u & T > T_m + \Delta T_u \end{cases} \tag{5}$$

$$\lambda = \begin{cases} \lambda_f & T < T_m - \Delta T \\ \lambda_f + \dfrac{\lambda_u - \lambda_f}{2\Delta T}[T - (T_m - \Delta T)] & T_m - \Delta T \le T \le T_m + \Delta T \\ \lambda_u & T > T_m + \Delta T \end{cases} \tag{6}$$

Using Eqs. (5) and (6), (1), (2), (3), and (4) can be written simply as

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) = C \frac{\partial T}{\partial t} \tag{7}$$

2.2. Boundary conditions and initial conditions

There are three kinds of common boundary conditions about the heat transfer problems. For the computational model of embankment in cold regions, in terms of the principle on the adhere layer (Zhu, 1988), the conditions at the fixed boundaries are

$$T|_{\Gamma_1} = T(t) \tag{8}$$

where Γ_1 is the upper boundary.

The conditions at the lateral boundaries, Γ_2, are

$$\left.\frac{\partial T}{\partial n}\right|_{\Gamma_2} = 0 \tag{9}$$

The conditions at the lower boundary, Γ_3, are

$$-\lambda \left.\frac{\partial T}{\partial n}\right|_{\Gamma_3} = q_w \tag{10}$$

where n is the external normal direction, and q_w is the heat flux.

Initial conditions are as follows

$$T_0 = T(x, y, t)|_{t_0} \tag{11}$$

2.3. Finite element equations

It is very difficult to obtain the analytical solution for the heat transfer problem. We obtain a solution by using a Galerkin method. The following FE formulae are obtained.

$$[K]\{T\}_t + [C]\left\{\frac{\partial T}{\partial t}\right\}_t = \{F\}_t \tag{12}$$

where $[K]$ is the stiffness matrix, $[C]$ is the capacity matrix, $\{T\}_t$ is the column vector of temperature, $\{F\}_t$ is the column vector of load, and t is the time.

Based on the backward difference method, Eq. (12) can be written as

$$\left([K] + \frac{[C]}{\Delta t}\right)\{T\}_t = \frac{[C]}{\Delta t}\{T\}_{t\text{-}\Delta t} + \{F\}_t \tag{13}$$

where Δt is the time step.

The calculation formulas of $[K]$, $[C]$ and $\{F\}_t$ were detailed introduced by Lewis et al. (1996). $[K]$, $[C]$ and $\{F\}_t$ are deterministic variables in the conventional deterministic FE analysis, so $\{T\}_t$ of Eq. (13) is a deterministic result. In this paper, $[K]$ and $[C]$ are not deterministic because soil properties are stochastic, and $\{F\}_t$ is not deterministic because upper boundary conditions are random. Therefore, $\{T\}_t$ of Eq. (13) is a random result.

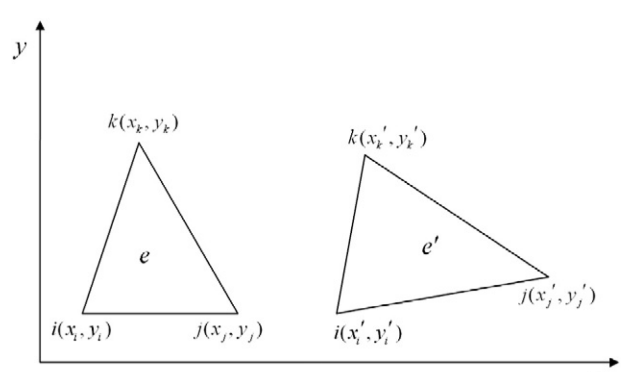

Fig. 1. The two-dimensional triangular random field elements.