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Maps for brittle and brittle-like failure in ice

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ABSTRACT

The strain rate and stress required for faulting in ice under low to moderate confinement are well understood and can be quantified in terms of independently measured materials parameters. Under high confinement, defined as confinement sufficient to suppress frictional sliding, brittle-like failure is still observed, even at pressure well below that required for faulting associated with phase transformations. While previous work has qualitatively suggested that high-confinement faults not associated with phase transformations, here termed P-faults, are likely associated with adiabatic instabilities, the failure map for P-faulting remains incomplete because of the lack of a quantitative understanding of the P-faulting terminal failure stress. Here we develop a new quantitative model for the P-faulting terminal failure stress that is consistent with recent experimental observations and then use this model to complete the failure map for compressive brittle and brittle-like failure of freshwater ice.

1. Introduction

The force on engineered structures in ice-infested waters is limited by loads induced by interactions with ice (ISO, 2010). Ice loads are often governed by compressive failure processes active under both high and low confinement (Kim et al., 2012) and thus a deeper understanding of the physical processes during compressive failure of ice can contribute to a better estimation of ice interaction loads. When ice is rapidly loaded under very little to no triaxial confinement, where confinement is defined below, compressive failure occurs via axial splitting (Wachter et al., 2008). Under low to moderate confinement failure during rapid loading occurs via shear faulting (Schulson et al., 1999). Because of the important role of friction in the formation of faults under low to moderate confinement (Renshaw and Schulson, 2001), we refer to low-confinement faults as Coulombic or C-faults. Under higher confinement ratios, frictional sliding is suppressed, but brittle-like failure is still observed, even at hydrostatic pressure well below that required for faulting associated with solid-state phase transformations (Kirby, 1987; Schulson, 2002). Failure is “brittle-like” in the sense that failure is accompanied by both the localization of strain along a macroscopic faulting plane and a sudden drop in the load-bearing ability of the material. Because of the important role of ductile deformation in the formation of high-confinement faults, we refer to high-confinement faults that occur in the absence of phase transformations as plastic or P-faults. By “plastic” in this context we mean von Mises or volume-conserving deformation, the resistance to which is independent of the hydrostatic component of the stress tensor (c.f. Schulson and Duval, 2009).

We define triaxial confinement, R_c, in terms of a ratio of the two principal stresses which have non-zero components when resolved onto the fault plane. The orientation of the fault plane, and thus which two principal stresses are used to define the confinement, depends on the microstructure of the ice. In granular ice (macroscopically isotropic microstructure), fault planes are oriented parallel to the direction of the intermediate principal stress. Because the fault plane is parallel to the direction of the intermediate principal stress, the component of the intermediate principal stress acting on the fault plane is zero. Thus R_c is defined as the ratio of the least to the greatest principal stress.

For S2 columnar ice (macroscopically isotropic only in the plane transverse to the long axis of the columnar-shaped grains), the orientation of the fault plane with respect to the microstructure varies depending on how the ice is loaded (Golding et al., 2010). Here we limit our attention to the case of most practical interest where the least principal stress acts in a direction parallel to the long axes of the columns, such as often occurs in horizontally-loaded sea ice. In this case under lower confinement fault planes are parallel to the long axes of the columns. Because the fault plane is parallel to the direction of the least principal stress, the component of the least principal stress acting on the fault plane is zero. Thus confinement is defined as the ratio of the intermediate to the greatest principal stress. Under higher confinement, faults may be oriented either parallel to the long axes of the columns, in which case confinement is defined in the same manner, or parallel to the direction of the intermediate principal stress, in which case confinement is defined as the ratio of the least to the greatest principal stress (see Figure 5 of Golding et al. (2010)).

Previous work has quantified the strain rate and stress required for C-faulting in ice and rock in terms of independently measured materials parameters (Renshaw and Schulson, 2001). Previous work has also defined the strains and strain rates necessary for P-faulting (Golding et al.,

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2012), again in terms of non-adjustable parameters. However, work to date has not quantified a failure-mode map, principally because our understanding of P-faulting remained incomplete. Here we develop such a map, based upon a new quantitative model for the P-faulting terminal failure stress that is consistent with recent experimental observations (Golding et al., 2010, 2012).

2. Model development

2.1. P-faulting

In P-faulting deformation is localized due to the instability that develops when thermal softening exceeds strain and strain rate hardening (Renshaw and Schulson, 2004). The requirements are twofold; a critical local effective strain ε_c^{local} is required to generate sufficient heat, and a critical local effective strain rate $\dot{\overline{\varepsilon}}_c^{local}$ is required to ensure approximately adiabatic conditions. By "local" we mean the strain and strain rate within the incipient plastic fault as opposed to the macroscopic "global" strain and strain rates obtained from measurements of displacement along boundaries. The use of local strain and strain rate permits us to relate fundamental materials properties to macroscopic constitutive relationships. Our use of "local" is not meant to imply the strain or strain rate at, for example, a particular node in a discretized finite element mesh. As a practical matter, in applying the results from this work, the strains and strain rates at discrete nodes are "global" even though they vary in both space and time; typically the local strain and strain rates are not modeled explicitly.

The twofold requirements for P-faulting can be quantified through a balance of the effects of strain and strain rate hardening and the effect of thermal softening (see eqns. 17.21 and 17.26 in Frost and Ashby, 1982)

$$\overline{\varepsilon}_c^{local} = \frac{-m\rho C_p}{\partial\overline{\sigma}/\partial T} \tag{1}$$

and

$$\dot{\overline{\varepsilon}}_c^{local} = \frac{2a\kappa\overline{\varepsilon}_c^{local}}{\rho C_p w_d^2} \tag{2}$$

where m is the work-hardening exponent in the expression $\overline{\sigma}\propto\overline{\varepsilon}^m$, where $\overline{\sigma}$ and $\overline{\varepsilon}$ are the effective stress and strain, respectively, $\dot{\overline{\varepsilon}}$ is the effective plastic strain rate, a is a geometrical factor of order unity, κ is the thermal conductivity, C_p is the specific heat, ρ is the density of ice, w_d is the characteristic length of heat diffusion, T is the temperature, and $\partial\overline{\sigma}/\partial T$ denotes thermal softening. Here effective stress, $\overline{\sigma}$, is defined for granular ice using von Mises' criterion for a plastically isotropic material; for columnar ice we use Hill's criterion (Hill, 1950) for a plastically anisotropic material (Golding et al., 2010). The effective strain is estimated from the inelastic strain increment, as determined for granular ice using the Levy–Mises relationship (Dieter, 1986) and for columnar ice using Hill's relationship (Hill, 1950). The local strain rate within the fault is related to measured rates of global effective deformation by the ratio of the volume of the fault to the volume of the ice, giving (Golding et al., 2012)

$$\dot{\overline{\varepsilon}}^{global} = \frac{\sqrt{2}w_f}{\ell}\dot{\overline{\varepsilon}}^{local} \tag{3}$$

where w_f is the width of the fault and ℓ is a characteristic length. In the laboratory where a single fault develops within sub-meter sized test specimens, ℓ is taken as the length of the specimen along the direction of shortening. Plastic strain is assumed equal to the inelastic strain.

The effective stress required for P-faulting can be estimated by first combining Eqs. (1)–(3) to give

$$\dot{\overline{\varepsilon}}_c^{global} = \frac{-2\sqrt{2}a\kappa m w_f}{\ell w_d^2(\partial\overline{\sigma}/\partial T)}. \tag{4}$$

This expression is then combined with a rate dependent plastic flow law to relate the critical global strain rate to the corresponding failure stress. Two phenomenological flow laws are commonly assumed to describe plastic deformation in ice. One is an Arrhenius power-law equation of the form

$$\dot{\overline{\varepsilon}}^{global} = A_p\overline{\sigma}^{n_p}e^{-Q_p/(RT)} \tag{5}$$

where A_p is a materials property that may depend on grain size and temperature, Q_p is the apparent activation energy, R is the gas constant, and n_p is the stress exponent. At relatively low stresses over the range of stresses where the power-law equation applies the stress exponent $n_p \sim 3$ (Frost and Ashby, 1982). We characterize this type of deformation as "low-stress power-law creep" or simply "low-stress creep". At higher stresses the creep rate limiting process is thought to transition from dislocation climb-controlled to dislocation glide-controlled flow (Frost and Ashby, 1982). We refer to this as "high-stress creep". Within the high-stress creep regime deformation is sometimes still described by an Arrhenius power-law equation (Eq. (5)), but with a stress exponent $n_p > 3$. Alternatively, the power-law relationship between effective stress and effective strain rate may break down in the high-stress creep regime and deformation may be instead characterized by an exponential flow law of the form

$$\dot{\overline{\varepsilon}}^{global} = A_e e^{\beta\overline{\sigma}}e^{-Q_e/(RT)} \tag{6}$$

where A_e and β are materials properties and Q_e is the apparent activation energy for high-stress creep. The transition from a low-stress power law to a high-stress exponential flow law can by quantified by a single hyperbolic creep law of the form (Garofalo, 1963; Wong and Jonas, 1968)

$$\dot{\overline{\varepsilon}}^{global} = A_h[\sinh(\alpha\overline{\sigma})]^{n_h}e^{-Q_h/(RT)} \tag{7}$$

where A_h is a materials property and the parameter α is the reciprocal of the effective stress at which low and high stress creep contribute equally to the global creep rate. For values of $\alpha\overline{\sigma}$ less than about 0.8, this expression reduces to the power-law creep expression (Eq. (5)) with $A_h = A_p/\alpha^n$. For $\alpha\overline{\sigma}$ greater than about 1.2 this expression reduces to the exponential flow law (Eq. (6)) with $\beta = n\alpha$.

2.2. C-faulting

The strain rate that marks the ductile–brittle transition under conditions that lead to C-faulting can be estimated following the analysis of Renshaw and Schulson (2001) and Schulson et al. (1999). Accordingly, and building on experimental observations, the analysis suggests that C-faulting initiates owing to the bending-induced failure of slender microcolumns created from sets of secondary cracks that emanate from one side of a sliding primary crack. The C-faulting transitional strain rate is then given by (Renshaw and Schulson, 2001)

$$\dot{\overline{\varepsilon}}_c^C = \frac{25K_{Ic}^{n_p}A_p e^{-Q_p/(RT)}}{d^{n_p/2}[(1-R_c)-\mu(1+R_c)]} \tag{8}$$

where $\dot{\overline{\varepsilon}}_c^C$ is the minimum strain rate required for C-faulting, K_{IC} is the fracture toughness, d is the grain size, μ is the friction coefficient, R_c is the confinement ratio of principal stresses (mentioned above), and the materials parameters A_p and Q_p are those for low-stress creep. In developing this model, Schulson et al. (1999) invoked an Arrhenius power-law relating stress to strain rate (Eq. (5)) and assumed $n_p = 3$.