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Snow load models for probabilistic optimization of steel frames

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ABSTRACT

Three probabilistic snow load models based on the Poisson spike process and the block maxima method are derived from the second moment probabilistic description of monthly extreme values of snow load. For comparison of the models, the upper tail asymptotic approximations of their cumulative probability distribution functions of yearly maxima are obtained. The performance of the models is checked by the probabilistic design and verification of representative portal frames of low-rise industrial buildings exposed to snow and wind loads using the first-order reliability method (FORM). The statistical parameters of monthly maxima of climatic loads related to measurements at six locations in Germany are employed. The considered snow load has a pattern of a seasonal occurrence of snowfall. The Poisson spike process model is then applied to the probabilistic optimization of safety factors for the standardized design of frames. A novel optimization of variable safety factors is proposed to differentiate design for frames with light to heavy weight roofs. It is shown that this differentiation significantly reduces scatter of the reliability level around the target value.

1. Introduction

Snow load models, intended for the probabilistic design of structures exposed to snow loads, have to be based on sufficiently long measurements of preferably daily snow water equivalent (SWE). SWE is often recorded in millimeters or kg/m^2; multiplying by $g = 9.81$ m/s^2 yields the snow load in N/m^2. At most stations with SWE records, the data are available only in longer intervals corresponding to e.g. weekly, decade or monthly measurements. Generally, less laborious daily measurements of snow depth, which are then on various levels of approximation transformed to the water equivalents, are taken. SWE can be derived from snow depth and snow density, but density may vary over a large range of 100 to 500 kg/m^3 (Sanpaolesi et al., 1998). Snow density appraisal for SWE estimation is a crucial issue also for hydrological models simulating runoff from a melting snowpack (e.g. Bocchiola and Groppelli, 2010).

For calibration of design procedures it is sufficient to conduct probabilistic studies and optimizations at a few representative stations only. These can be selected also with respect to the quality of snow load records. The minimal length of the records is usually related to the definition of the characteristic snow load on the ground. In Eurocode EN 1990: 2002 "Basis of structural design" the characteristic value is determined on the basis of the annual probability of exceedance of 0.02, i.e. corresponding to a mean return period of 50 years. Thus, the records should cover at least of the order of 40 to 50 years (winter seasons) (Sanpaolesi et al., 1998). In conjunction with a common 50 year life time of buildings (EN 1990: 2002), snow load records of comparable length are deemed sufficient for the determination of distribution functions to be used in probabilistic studies of ultimate limit states verification for persistent and transient design situations. For accidental design situations due to natural hazards, events with a 10,000 year mean return period should be predicted and special approaches are needed (e.g. Sadovský et al., 2012).

The present study employs second moment statistical characteristics related to the Gumbel distribution of monthly extremes of snow load on the ground and the 10 minute mean wind pressure at a height of 10 m above flat open terrain, as well as the average percentage of month with snow at six locations in Germany (Schleich et al., 2002). Such data can be commonly obtained in climates with seasonal snow occurrence. Based on these characteristics three snow load models are considered for optimization study. The models are compared by the upper tail asymptotic approximations of their cumulative probability distribution functions of yearly maxima. The performance of the models is checked by the probabilistic design and verification of representative steel portal frames of low-rise industrial buildings exposed to snow and wind loads. For probabilistic calculations, the first-order reliability method (FORM) is applied (e.g. Melchers, 2001).

The most general snow load model allowing advanced treatment of detailed measurement records (when available, e.g. daily SWE) is used for probabilistic optimization of the safety factors for the standardized design of frames. Actual dimensions of the frames, limit state functions and probabilistic descriptions of material properties and climatic loads are adopted from the work of Schleich et al. (2002). An extension of the design situations for optimization across light to heavy weight

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roofs is taken from Sadovský and Páleš (2008). The original optimization aiming at unified standardized design of frames is extended to optimization of variable safety factors. The goal of the presented novel optimization is a differentiated design for frames with light to heavy weight roofs resulting in reliability levels better approximating the target value, thereby improving both the reliability and economy of the standardized design.

This study intends to contribute to the discussion platform on the Evolution of Eurocodes, which is currently being prepared under the auspices of the technical subcommittee CEN/TC 250/SC1 "Eurocode 1: Actions on structures". Particularly, the query on the current values of partial factors for the climatic actions $\gamma_Q = 1.5$ and the load combination factors for snow $\psi_{0S} = 0.5$ and wind $\psi_{0W} = 0.6$ are addressed.

The snow load models are described in Section 2. Section 3 shows the numerical results of probabilistic design of the representative steel frames as well as the verification of the standardized design of the frames applying the current values of partial factors and combination factors for climatic loads. Probabilistic optimization of the standardized safety factors is carried out in Subsection 4.1. Optimization of variable safety factors is treated in Subsection 4.2. Discussion section and Conclusions include practical implications and recommendations for further research and calibration studies.

2. Snow load models

The considered snow load models are aimed at application in FORM probabilistic calculations. FORM is an approximate method for a multi-dimensional integration of a joint probability density function of the vector X of several continuous random variables. In the structural reliability analysis, the integration domain is specified by the so-called limit-state function separating the domains of failure and reliability by the failure surface. For the assessment of probability of failure the random variable arguments X are transformed into independent standardized normal variables U. For independent random variable components X_i, the transformation into U_i simplifies to marginal distributions $F_{Xi}(x_i) = \Phi(u_i)$, Φ denoting the standardized normal distribution function. The shortest distance from the origin to the limit-state surface in the transformed space represents the geometric reliability index β yielding the FORM probability of failure by the expression $P_f = \Phi(-\beta)$. The point on the failure surface realizing the shortest distance is called the design point. It is the most likely point on the failure surface. The FORM P_f value is an exact probability of failure for a tangent hyperplane approximating the failure surface at the design point in the transformed space. For details see e.g. Melchers (2001).

For FORM application the time dependent processes like climatic loads are represented by time invariant random variables. Two different approaches can be used. The first one is the block maxima method. Taking the maximum values within the year or month the well developed theory of extreme value distributions of independent equally distributed random variables can be applied (Coles, 2001; Reiss and Thomas, 2007). The three-parameter generalized extreme value distribution includes the Gumbel, Frechet and Weibull distributions. For snow loads a two-parametric Gumbel distribution is broadly accepted (Sanpaolesi et al., 1998). Reiss and Thomas (2007) have pointed out that by dealing with annual maxima one is avoiding the problems with serial correlation and seasonal variation, but loses information contained in the data. Therefore, a first modification of this approach would be to base the inference on seasonal or monthly maxima.

The second approach efficiently uses maxima of independent events. Provided the events occur with independent counts on non-overlapping subsets the most general approach is modeling the event maxima by the Poisson process (Coles, 2001; Reiss and Thomas, 2007; Melchers, 2001). The distribution function of event maxima follows from the probability of no maxima above a given level in the considered time interval.

The employed probabilistic description of climatic loads (Schleich et al., 2002) is based on the Gumbel distribution of their monthly extremes (assuming occurrence of the snow load) $F_Y(y)$:

$$F_Y(y) = \exp\left\{-\exp\left[-\frac{\pi}{\sigma\sqrt{6}}(y-\mu)-\gamma\right]\right\}, \quad \gamma \cong 0.577216 \tag{1}$$

where μ is the mean and σ standard deviation. The inverse function of $F_Y(y)$ is:

$$y = \mu\left\{1-\frac{\sigma\sqrt{6}}{\mu\pi}[\gamma + \ln(-\ln F_Y(y))]\right\}. \tag{2}$$

For considering the snow load seasonal occurrence, the probability of a month with snow p_S is specified by Schleich et al. (2002), see Table 3.

The first snow load model adapts the Poisson process to the considered probabilistic description of snow loads. The second and third models are based on the block maxima method. The difference between the models is in implementation of the seasonal snow load occurrence.

2.1. The model based on the Poisson spike process

Consider the maximum of daily snow loads resulting from a snow cover continuing from the first snowfall, possible accumulation of snow covers and melting up to depletion. Referring to the available parameters of monthly extremes, it is assumed that the number of occurrences of the maxima per year (including the whole winter season) corresponds to the number of months with snow. The counts of months with snow in a year are random variables with the mean rate of occurrence given by $\nu = 12p_S$. They may be assumed as independent between any two months, which means that the maxima occurrences can be studied by the Poisson counting process.

Due to the relatively short durations, snow load maxima are considered as independent random spike loads with probability distribution given by Eq. (1). The random time of occurrences of the spikes, independent between individual pulses, and their random magnitudes describes the Poisson spike process, cf. Melchers (2001). Then the up-crossing rate for a given level a per time unit is

$$\nu_a^+ = F_Y(a)[1-F_Y(a)]\nu \cong [1-F_Y(a)]\nu \tag{3}$$

since $F_Y(a)$ is close to one for high levels of snow loads used in structural design. The probability of the first level up-crossing in the time interval [0, t] is then

$$p_f(t,a) = 1-\exp\left[-\nu_a^+ t\right]. \tag{4}$$

The corresponding probability distribution of snow load maxima is

$$F_{X\max(t)}(x) = 1-p_f(t,x) = \exp\{-[1-F_Y(x)]\nu t\}. \tag{5}$$

For FORM calculations the inverse of the functional relationship in Eq. (5) is useful

$$F_Y(x) = 1+\frac{1}{\nu t}\ln\left[F_{X\max(t)}(x)\right] \tag{6}$$

resolving the transformation $F_{X\max(t)}(x) = \Phi(u)$ for u from the standardized normal space by Eq. (2).

From Eq. (5) the asymptotic approximation of the upper tail of the distribution of yearly snow load maxima can be readily expressed in terms of the small quantity $\Delta(x) = 1 - F_Y(x)$ as

$$F_{X\max(t=1)}(x) \cong 1-\nu\Delta x. \tag{7}$$