Cold Regions Science and Technology 85 (2013) 256–260

Contents lists available at SciVerse ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

A model of hoarfrost formation on a cable

Lasse Makkonen *

VTT Technical Research Centre of Finland, Box 1000, 02044 VTT, Espoo, Finland

ARTICLE INFO

Article history:
Received 27 June 2012
Accepted 1 October 2012

Keywords:
Hoarfrost
Frost
Icing
Ice accretion
Ice growth
Power line
Corona loss

ABSTRACT

A time-dependent numerical model of hoarfrost formation on an overhead cable is presented. The model is aimed at calculating the thickness of a hoarfrost layer using routinely measured meteorological data as input. The growth rate and density of hoarfrost are simulated. This requires continuous calculation of the surface temperature around the cable. The model also simulates the disappearance of hoarfrost by sublimation, melting and dropping off. The feasibility of explaining the occurrence of corona losses on overhead power transmission cables by the modeled hoarfrost thickness is demonstrated by field data.

1. Introduction

Hoarfrost forms when water vapor changes directly into solid ice. Under natural atmospheric conditions the resulting ice deposition consists of loosely spaced needle-like thin ice crystals (Fig. 1). The dimensions of hoarfrost deposits are usually small and the density of them is very low. Hoarfrost is easily blown off by the wind. Consequently, hoarfrost alone does not cause ice loads or aerodynamic effects that are significant to structural safety, unlike rime, glaze and wet snow (Makkonen, 2000).

Nevertheless, hoarfrost causes problems, particularly on overhead cables on which it is related to corona discharges and consequent power losses (Lahti et al., 1997). Corona losses in power transmission are caused also by precipitation, but their magnitude is at its greatest when the electric field at the conductor surface is enhanced by the presence of the ice needles of a hoarfrost deposition. This phenomenon causes electricity transmission losses that are of significant economic value in cold regions (Sollerkvist et al., 2007). Furthermore, on overhead cables for trains and trolleys, hoarfrost causes a corona discharge at the contact with the pantograph, causing excess wear and a light and noise problem (Kamata et al., 2012).

During hoarfrost formation in nature the humidity of air is not necessarily high. The outgoing long wave radiation may cool the surface so much that deposition occurs even at a relatively low humidity. When the air is so humid that it is saturated with respect to ice, deposition occurs even without the radiation cooling effect. Thus, hoarfrost may form simultaneously with in-cloud icing. Intensive icing due to accreting droplets may then occur and the portion of the icing rate due to vapor deposition is typically small. However, the rate of hoarfrost formation is approximately proportional to the surface area, whereas the rate of rime icing increases more slowly with increasing object size. Therefore, for ice load modeling on very large objects hoarfrost may need to be taken into account. Moreover, very accurate modeling of rime icing, required by the rotating multi-cylinder method to measure cloud liquid water and droplet size, must include vapor deposition (Makkonen, 1992).

Hence, there are several applications in which numerical modeling of hoarfrost formation under natural conditions is useful. The best prospect in utilizing hoarfrost modeling is in the significant savings to be achieved if hoarfrost on cables could be predicted using a weather forecasting model and reduced by controlled Joule heating. Here, a physical–numerical model of hoarfrost is proposed. In contrast to some other models of frost formation on planar surfaces and in industrial applications (Hayashi et al., 1977; Kandula, 2011; Mago and Sherif, 2005; Östin and Andersson, 1991; Raju and Sherif, 1992; Saito et al., 1984; Schneider, 1977; Seki et al., 1984), this model is developed specifically for simulating hoarfrost thickness on a cable in a natural outdoor environment.

2. The model

The foundation of this hoarfrost formation model is the cylinder icing model by Makkonen (1984), discussed also in Makkonen (2000) and Poots (1996). The changes made to the icing model for this study on hoarfrost are:

1. A sub-program simulating vapor deposition rate and hoarfrost density is included, as will be explained below.

* Tel.: +358 20 722 4914; fax: +358 20 722 7007.
E-mail address: lasse.makkonen@vtt.fi.

0165-232X/$ – see front matter
http://dx.doi.org/10.1016/j.coldregions.2012.10.001

Nomenclature

C	corona loss in electric power transmission, MW
c_c	specific heat of the cable material, J/(kg K)
c_p	specific heat of air, J/(kg K)
d	cable diameter, m
D	iced cylinder diameter, m
e_a	water vapor pressures in air, Pa
e_s	water vapor pressure over ice, Pa
g	gravitational constant, m/s^2
Gr	Grashof number
H	mean thickness of the simulated hoarfrost deposit, mm
h	convective transfer coefficient, $h = k_a Nu/D$, W/(m^2 K)
I	rate of ice formation, kg/(m^2 s)
k_a	heat conductivity of air, W/(m K)
k_i	heat conductivity of the simulated hoarfrost deposit, W/(m K)
L_e	latent heat of sublimation of ice at t_s, J/Kg
n	cloudiness, parts of ten
Nu	Nusselt number
Nu_v	Nusselt number in free convection
Nu_b	Nusselt number in forced convection
p_a	atmospheric pressure, Pa
q_c	flux of sensible heat to air, W/m^2
q_e	heat flux due to release of latent heat of sublimation, W/m^2
q_{eff}	effective outgoing long wave radiation flux, W/m^2
$q_{eff,0}$	outgoing long wave radiation flux at clear skies, W/m^2
q_i	conductive heat flux onto the ice surface through ice, W/m^2
q_S	effective Sun's short wave radiation flux, W/m^2
Q_J	power due to Joule heating of the cable, W/m
Q_t	power due to thermal inertia of the cable, W/m
Re	Reynolds number, vD/υ
t_a	air temperature, °C
T_a	air temperature, K
t_c	cable temperature, °C
T_c	cable temperature, K
t_s	ice surface temperature, °C
T_s	ice surface temperature, K
v	wind speed, m/s

Greek symbols

υ	kinematic viscosity of air, m^2/s
σ	Stefan–Boltzmann coefficient, J/(m^2 K^4)
ρ	density of the forming hoarfrost, kg/m^3
ρ_c	density of the cable material, kg/m^3
ρ_t	simulated overall hoarfrost density, kg/m^3
ΔT_c	decrease in cable temperature during a model time-step, K
$\Delta\tau$	time-step in the model, s

2. Free convection is taken into account in the heat balance because hoarfrost may form when there is no wind.
3. The outgoing long wave radiation and Sun's direct radiation are included in the heat balance since hoarfrost may form during clear skies.
4. The heat balance is calculated separately on the windward side and the lee side, and on the upper side and lower side of the object.
5. Heat transfer from the conductor to the surface of ice is modeled. This allows including the effects of the Joule heating and cable thermal inertia.
6. The heat balance terms related to droplet impingement are neglected since this model is for simulating hoarfrost only. A necessary input then is the humidity of air.
7. Ice disappearance is modeled by including evaporation and melting, as well as a criterion of ice release by shedding. This makes long-term continuous modeling possible.

The model simulates the mean rate of ice accretion, i.e. the deposition rate, and the mean thickness of the accretion around the cable. They are based on modeling icing at four sections around a horizontally oriented cable. This requires solving numerically the heat balance and surface temperature of these sections, as discussed in the following.

2.1. Deposition

The vapor deposition rate is I calculated by Eq. (1). When I is negative, sublimation of ice occurs (when the simulation shows ice on the section surface).

$$I = \frac{0.622}{c_p p_a} h(e_s - e_a). \tag{1}$$

Here c_p is the specific heat of air, p_a the atmospheric pressure, h the convective transfer coefficient and e_s and e_a the water vapor pressures over ice and in air, respectively. The convective transfer coefficient h depends mainly on wind speed and cable diameter as well as surface roughness, see below and Makkonen (1985). The vapor water pressure in air, e_a, is an input parameter for the modeling. It is typically obtained by measuring the relative humidity and temperature in air. The equilibrium water vapor pressure over ice, e_s, depends on the surface temperature of the ice, t_s, the modeling of which is, therefore, a critical issue here. Moreover, Eq. (1) only applies when $t_s < 0$ °C.

2.2. The heat balance

The surface temperature of the ice, T_s, is solved from the heat balance of the section of the cable by numerical iteration.

With the assumptions explained above the heat balance in the model is

$$q_e + q_s + q_i = q_c + q_{eff} \tag{2}$$

where the heat flux terms are explained in the nomenclature and discussed in the following.

2.2.1. Convective heat transfer

The heat flux due to release of latent heat of phase change from vapor to ice is

$$q_e = L_e I \tag{3}$$

where L_e is the latent heat of sublimation of ice at t_s.

The heat flux due to convective heat transfer to air is

$$q_c = h(T_s - T_a). \tag{4}$$

Both q_e and q_c depend on the convective heat transfer coefficient

$$h = k_a Nu/D \tag{5}$$

which is calculated separately on the upper side and lower side of the cable, and in the presence of wind separately on the windward and the leeward surfaces. Here k_a is the heat conductivity of air, D is the iced cylinder diameter and Nu is the Nusselt number.

In free convection the Nusselt number is calculated by Sparrow and Stretton (1985)

$$Nu_v = 0.395\ Gr^{0.25} \tag{6}$$