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A mechanism for mode selection in melt band instabilities

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ABSTRACT

The deformation of partially molten mantle in tectonic environments can lead to exotic structures, which potentially affect both melt and plate-boundary focussing. Examples of such structures are found in laboratory deformation experiments on partially molten rocks. Simple-shear and torsion experiments demonstrate the formation of concentrated melt bands at angles of around 20° to the shear plane. The melt bands form in the experiments with widths of a few to tens of microns, and a band spacing roughly an order of magnitude larger. Existing compaction theories, however, cannot predict this band width structure, let alone any mode selection, since they infer the fastest growing instability to occur for wavelengths or bands of vanishing width. Here, we propose that surface tension in the mixture, especially on a diffuse interface in the limit of sharp melt-fraction gradients, can mitigate the instability at vanishing wavelength and thus permit mode selection for finite-width bands. Indeed, the expected weak capillary forces on the diffuse interface lead to predicted mode selection at the melt-band widths observed in the experiments.

1. Introduction

While mantle melting only occurs within a small volume of the Earth, it plays a disproportionate role in both geochemical evolution and plate-boundary processes (see Cox et al., 1993). Indeed, the unique deformation of partial melts likely controls flow and strain focussing at both convergent and divergent plate boundaries (e.g., Spiegelman and McKenzie, 1987; Katz, 2008; Gerya and Meilick, 2011; Gerya, 2013). In particular, sheared partial melts have been demonstrated in laboratory experiments (Daines and Kohlstedt, 1997; Holtzman et al., 2003; King et al., 2010; Qi et al., 2013) to develop narrow melt bands at shallow angles (∼20°) to the direction of motion. Such melt banding may provide high-permeability pathways that strongly influence the transport of melt to the Earth's surface (Kohlstedt and Holtzman, 2009).

The observed shallow angle of these melt bands is enigmatic and has been the subject of several theoretical studies invoking two-phase compaction theory with various rheological mechanisms (Stevenson, 1989; Spiegelman, 2003; Katz et al., 2006; Takei and Holtzman, 2009; Butler, 2012; Takei and Katz, 2013; Katz and Takei, 2013; Rudge and Bercovici, 2015). An equally significant enigma is that current two-phase models cannot predict the basic melt band width, since they infer the fastest growing instability to have zero wavelength. Laboratory experiments, however, show that while the melt bands are very narrow, of order a few to tens of microns wide, and with band spacing roughly an order of magnitude wider (Holtzman et al., 2003; Holtzman and Kohlstedt, 2007; Kohlstedt and Holtzman, 2009), they are consistently not vanishingly small. The failure to predict mode selection has been a significant conundrum for understanding the physics let alone believing the theories, and is problematic for numerical simulations for which instabilities shrink to the grid-scale, and thus cannot be resolved (Katz et al., 2006; Butler, 2012; Alisic et al., 2014). Butler (2010) proposed that, in the finite strain limit, the rotation of bands through the optimal angle of growth can amplify larger wavelength bands, although as shown earlier by Spiegelman (2003) this effect depends on the initial conditions for the structure of the porosity perturbations. Takei and Hier-Majumder (2009) proposed that compaction coincident with dissolution and precipitation provides mode selection governed by a chemical diffusion length scale, which is indeed similar to the widest band spacing, although not the band widths. However, while such chemical reactions between phases are expected to be important in geological settings (Aharonov et al., 1997), their role was not evident in the laboratory experiments, which were designed to study melt channels by stress alone and avoid reaction channelization (Holtzman et al., 2003).

Here we consider two-phase compaction theory that includes capillary effects from the interface between phases, i.e., the melt

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and matrix (Stevenson, 1986; Bercovici et al., 2001; Bercovici and Ricard, 2003; Hier-Majumder et al., 2006), as a means for explaining mode selection. However, capillary effects at the microscopic (i.e., pore/grain) scale cannot give the necessary effect (as will be demonstrated herein). In this paper, we propose a small adjustment to existing theories that involves a diffuse interface effect, which occurs at very large gradients in melt volume fraction (e.g., Sun and Beckermann, 2004). Below we briefly develop the concept of the diffuse interface coincident with microscopic interfaces, and demonstrate how it can predict mode selection at the observed melt-band wavelengths.

2. Theory

2.1. Two-phase mixture interface and diffuse interface

Various two-phase flow theories treat the interface between phases and associated surface energy and surface tension by defining an interface area density (i.e., interface area per unit volume) α (see Ni and Beckerman, 1991; Bercovici et al., 2001). For example, if a volume δV of mixture is filled with N spherical fluid bubbles of radius r, surrounded by an opposite matrix phase, then the fluid volume fraction is $\phi = N\frac{4}{3}\pi r^3/\delta V$, while $\alpha = N4\pi r^2/\delta V$; in the same vein, the average curvature of this interface would be $d\alpha/d\phi = (d\alpha/dr)/(d\phi/dr) = 2/r$ as expected.

However if the mixture has sharp gradients in fluid fraction $\nabla\phi$, then the gradient region itself can appear as an effective or diffuse interface. Sun and Beckermann (2004) consider a diffuse interface in a mixture and invoke the formalism of phase-field theory (Anderson et al., 1998; Chen, 2002; Moelans et al., 2008) to propose an adjusted model for interface density and curvature. We appeal to some of their concepts but diverge in other respects. One important deviation is that phase-field theory only has interfaces defined by gradients in the phase variable, while we have both a background interface from a more homogeneous distribution of phases (i.e., bubbles and grains) in addition to an effective diffuse interface caused by sharp gradients in the fluid volume fraction.

Although the interface density α may be affected by a diffuse interface, only the curvature appears in the dynamics and thus we need only specify how $d\alpha/d\phi$ is altered. Indeed as shown in Appendix A, we infer an effective curvature

$$\frac{d\alpha}{d\phi} = \frac{d\mathcal{A}}{d\phi} - \frac{1}{\mathcal{A}}\nabla^2\phi \tag{1}$$

where $\mathcal{A}$ is the microscopic (pore and grain) scale interface area, which we assume is only a function of porosity (see Bercovici et al., 2001; Hier-Majumder et al., 2006). The two terms on the right of (1) are due to microscopic scale interface curvature originally described by Bercovici et al. (2001) (first term), and that due to sharp coherent structures in the porosity field (second term). For example, a coherent structure with a sharp gradient in porosity can resemble a macroscopic bubble wall separating low and high porosity regions, which then has a net effective surface tension on it. However, there is a continuum of coherent structures between weak gradients for which the diffuse interface will barely register, to sharper ones. Indeed, since $\mathcal{A}$ is a large zeroth-order term, the diffuse interface curvature term only becomes important for sharp gradients in ϕ. Equation (1) is the same as the mean curvature inferred by Sun and Beckermann (2004), however we diverge from those authors by retaining (1) as the full effective interface curvature, while they argue to remove the microscale curvature, i.e., the first term on the right of (1). We retain this term since it is responsible for driving phase self-separation and/or wetting. Indeed the 2nd term retards self-separation once the porosity gradients get very large, and leads to a steady state separation rather than run-away separation. But to allow initial capillary effects on the pore or grain scale, we retain the micro-scale curvature term. In the end, the new effective interface curvature $d\alpha/d\phi$ can be employed in the appropriate two-phase theory (Bercovici et al., 2001; Bercovici and Ricard, 2003).

2.2. Mass conservation

Conservation of mass in two-phase continuum mechanics dictates a relation for the volume fraction ϕ_i of phase i (i.e., either phase), which, assuming both phases are incompressible and there is no mass exchange between phases, leads to

$$\frac{\partial\phi_i}{\partial t} + \nabla\cdot(\phi_i\mathbf{v}_i) = 0 \tag{2}$$

where $\mathbf{v}_i$ is the velocity of phase i. Summing these equations and noting that $\sum_i \phi_i = 1$, we arrive at

$$\nabla\cdot\bar{\mathbf{v}} = 0 \tag{3}$$

where $\bar{\mathbf{v}} = \sum_i \phi_i\mathbf{v}_i$. We can also define the unsubscripted $\phi = \phi_1$ as the volume fraction of the minor phase, here the fluid or melt phase. We also define the unsubscripted $\mathbf{v} = \mathbf{v}_2$ as the velocity of the solid or matrix phase, and $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$ as the phase separation velocity. We can hence recast (2) and (3) as

$$\frac{D\phi}{Dt} = (1-\phi)\nabla\cdot\mathbf{v} \tag{4}$$

and

$$\nabla\cdot\bar{\mathbf{v}} = \nabla\cdot(\mathbf{v} - \phi\Delta\mathbf{v}) = 0 \tag{5}$$

where $D/Dt = \partial/\partial t + \mathbf{v}\cdot\nabla$ is the material derivative in the matrix frame of reference.

2.3. Dynamics

The conservation of momentum in a creeping two-phase medium is generally prescribed (following Bercovici and Ricard, 2003, 2012)

$$\begin{aligned} 0 = &-\phi_i\nabla\Pi_i + \nabla\cdot(\phi_i\underline{\boldsymbol{\tau}}_i) \pm c\Delta\mathbf{v} \\ &+ \omega_i\left(\Delta\Pi\nabla\phi + \nabla(\gamma\alpha)\right) \end{aligned} \tag{6}$$

where we neglect gravity for the application at hand, and where the internal pressure on phase i is Π_i, $\underline{\boldsymbol{\tau}}_i$ is the deviatoric stress tensor in phase i, phase density is ρ_i, c is the coefficient of drag between phases, $\Delta\Pi = \Pi_2 - \Pi_1$, γ is the surface tension on the interface between phases, α is again the interface density, and ω_i is a weighting factor (such that $\sum_i \omega_i = 1$) that accounts for how much surface tension is embedded in one phase relative to the other.

2.3.1. Constitutive laws and rheology

Since phase 1 is a melt we assume $\underline{\boldsymbol{\tau}}_1 \approx 0$ and $\omega_1 = 0$ (Bercovici and Ricard, 2003). The matrix deviatoric stress is thus denoted as $\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\tau}}_2$ and given by

$$\underline{\boldsymbol{\tau}} = 2\mu\underline{\dot{\boldsymbol{\varepsilon}}} = \mu\left(\nabla\mathbf{v} + [\nabla\mathbf{v}]^{\mathrm{t}} - \frac{2}{3}\nabla\cdot\mathbf{v}\underline{\boldsymbol{I}}\right) \tag{7}$$

where μ is the matrix viscosity, and $\underline{\dot{\boldsymbol{\varepsilon}}}$ is the matrix deviatoric strain-rate tensor, $[..]^{\mathrm{t}}$ implies tensor transpose and $\underline{\boldsymbol{I}}$ is the identity tensor. In keeping with prior analysis (Katz et al., 2006), we allow that $(1-\phi)\mu$ is an effective viscosity given generally by

$$\begin{aligned} \mu_{\mathrm{eff}} &= (1-\phi)\mu = \mu_0\Lambda\left(\phi, \dot{\varepsilon}^2\right) \\ &= \mu_0 e^{-b(\phi-\phi_0)}\left(\frac{\dot{\varepsilon}^2}{\dot{\varepsilon}_0^2}\right)^{\frac{1-n}{2n}} \end{aligned} \tag{8}$$