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The Earth's free core nutation: Formulation of dynamics and estimation of eigenperiod from the very-long-baseline interferometry data

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ABSTRACT

The free-core nutation (FCN) is a rotational normal mode of the Earth's outer core. We derive the equations of motion for FCN w.r.t. both the inertia space $\boldsymbol{F_0}$ and the uniformly rotating frame $\boldsymbol{F_\Omega}$, and show that the two sets of equations are invariant in form under the reference frame transformation, as required by physics. The frequency-domain formulation describes the FCN resonance (to nearby tidal signals), which has been exploited to estimate the complex eigenfrequency of FCN, or its eigenperiod P and quality factor Q. On the other hand, our time-domain formulation in terms of temporal convolution describes the response of the free FCN under a (continual) excitation. The convolution well explains the dynamic behaviors of FCN in the observed very-long-baseline interferometry (VLBI) data (in $\boldsymbol{F_0}$), including the undulation of the FCN amplitude and the apparent fluctuations in the period and phase over time, as well as the temporal concurrence of a large phase jump with the near-zero amplitude during ~1998–2000, in complete analogy to the observed behavior of the Chandler wobble (in $\boldsymbol{F_\Omega}$). The reverse, deconvolution process is further exploited to yield optimal estimates for FCN's eigenfrequency using the VLBI data, following the approach of Furuya and Chao (1996) of locating minimum excitation power. While this method is found to be insensitive to Q owing to the short timespan of the data, we obtain the estimate of $P = 441 \pm 4.5$ sidereal days (sd) where the 1-sigma uncertainty is assessed via extensive Monte Carlo simulations. This value is closer to the theoretical value of ~460 sd predicted by Earth models assuming hydrostatic equilibrium than do the prior estimates (425–435 sd) by the resonance method. The deconvolution process also yields the excitation function as a by-product, the physical sources of which await further studies.

1. Introduction

The classical astronomical precession–nutation of the Earth's rotational axis is driven by the luni-solar tidal torques exerted on the oblate, quasi-rigid Earth (e.g., Melchoir, 1983; Wahr, 1981a). In parallel, the Earth has a rotational normal mode known as the free-core nutation (FCN), a retrograde motion (clockwise as viewed from north) of the misalignment of the rotation axis of the spheroidal fluid outer core w.r.t. the figure axis of the spheroidal solid mantle (Toomre, 1974; Smith, 1977; Wahr, 1981b). Both the astronomical nutations and FCN have periods much longer than one day w.r.t. the inertial space, or near-diurnal periods w.r.t. the rotating Earth.

The FCN is just one of Earth's rotational modes which also include the better-known Chandler wobble (CW), along with those supposedly belonging to the solid inner core (the so-called prograde free-core nutation and the inner core wobble) (e.g., Mathews et al., 2002; Dehant and Mathews, 2007). In this sense the nutations can be regarded as the rotational response of the FCN-resonance system, just as the polar motion is regarded as the rotational response of the CW-resonance system, to various astronomical and geophysical forcings. The nutation terms are driven by the discrete-frequency luni-solar tidal torques but modified in amplitude and phase by the FCN resonance; only the few of these tidal components at periods in close proximity to that of FCN are modified (amplified) to an appreciable extent.

The FCN itself may or may not appear in actual observations, depending on whether and how strongly it is actually excited by pertinent geophysical excitation mechanisms whatever they may be. The very-long-baseline interferometry (VLBI) technique has been measuring the Earth's nutations since the early 1980s.

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By the 1990s, as the VLBI data accumulated and continually improved in precision, a significant FCN signals began to emerge with varying amplitudes as large as 0.1–0.5 milliarcsecond (mas) peak-to-peak (Herring and Dong, 1994; Shirai and Fukushima, 2001; Herring et al., 2002; Lambert and Dehant, 2007).

In this paper we shall study the FCN signal in the VLBI nutation data in two fronts: First, we shall formulate the physics of FCN, that of a forced 2-D simple harmonic motion in a uniformly rotating reference frame. It takes the form of a temporal convolution of the resonance with some excitation function. We do not inspect the identity and behavior of the excitation process itself. Rather we grant the *existence* of the excitation for the FCN, and show that the physics of the convolution well explains the general behavior of the FCN observed in the VLBI data.

Secondly, exploiting the convolution formulation we shall estimate from the VLBI data the FCN's complex eigenfrequency, i.e. its natural period (eigenperiod) and the decay rate or quality factor. These are gross Earth data that contain important information about the property of the core and core–mantle interactions (e.g., Mathews et al., 2002; Dehant and Mathews, 2007). Prior estimates of the eigenperiod (w.r.t. to inertia frame) by means of the resonance method clustered around 425–435 sidereal days (sd) (see e.g. Rosat et al., 2009, for a review). That is significantly shorter than the 460 sd predicted by idealized Earth models under the assumption of rotation-gravitational hydrostatic equilibrium (Sasao et al., 1980; Wahr, 1981b). That suggests a core configuration that is significantly more oblate than the hydrostatic equilibrium (Gwinn et al., 1986), presumably under the influence of certain core–mantle torquing mechanism yet unattained in numerical or physical modeling (Buffett et al., 2002). Meanwhile, the estimates of FCN's quality factor range upwards from a few thousand but remain poorly constrained. Here we shall revisit this subject and, alternative to the resonance method, we estimate the FCN's eigenfrequency by the deconvolution approach of Furuya and Chao (1996); see also Gross (2007). We reach optimal estimates of the FCN eigenperiod which lie between the prior estimates and the theoretical value.

2. VLBI data

Referencing to distant celestial quasars, the VLBI technique measures, among other things, the 3-D Earth rotation parameters starting in the early 1980s, nowadays achieving accuracies better than ∼0.1 mas (IERS Conventions, 2010). As part of the Earth rotation parameters, the 2-D nutational motion in the Earth's rotation axis orientation in space is customarily given in terms of the celestial pole offsets $d\psi$ and $d\varepsilon$, i.e. the *deviations* of the longitude ψ and the obliquity ε of the equator in the ecliptic coordinates, referenced to the model values.

The VLBI nutation $d\psi$ and $d\varepsilon$ data used presently are the combined EOP 08 C04 data series from the International Earth Rotation and Reference Systems Service (IERS). The data are referenced to the IAU 2006/2000A precession–nutation model (e.g. Wallace and Capitaine, 2006) consistent with ITRF2008 reference system (Bizouard and Gambis, 2009) and adopted by IERS Conventions 2010 based on and updated from Mathews et al. (2002). The IAU2000A reference model accounts for all the nutation terms considered to be the Earth's response to the luni-solar tidal forcings (where FCN resonances are considered), whereas the physical parameters that are poorly known yet relevant to the rotation are estimated to best match the VLBI data. The FCN signal itself is left intact.

The VLBI data as provided have been homogenized and slightly smoothed to nominal intervals of 1 solar day. Our analysis is solely based on post-1992 data for their better quality, spanning 23 years of 1992–2014, long enough to resolve spectrally the FCN from nearby tidally driven terms. We shall refer to this VLBI data series as the "full dataset", whereas any segment thereof as n-year segment dataset. Only for comparison purposes will we present the rather noisy pre-1992 data and results derived from them.

To ensure the "cleanness" of the FCN signal we remove any residual tidal terms as well as the seasonal terms of non-tidal meteorological origin (after editing out obvious out-lier points). We do so by the linear least-squares regression (on the full dataset) and subtraction of the following periodic terms: the major tidal terms of Mf (13.6608 days), Msf (14.7653 days), Mm (27.5546 days), 9.31 year and 18.6 year, two seasonal (annual and semi-annual) terms, and a long-term linear trend accounting for any unmodeled precession. The removed terms are actually quite small, making no appreciable differences.

Fig. 1 gives the "cleaned" FCN time series of $d\psi$ and $d\varepsilon$, along with the time-frequency wavelet spectrum of the complex quantity $m(t) = \sin\varepsilon_0 d\psi(t) + i d\varepsilon(t)$ (see Equation (5) below) calculated adopting the Morlet wavelet, a normalized Gaussian-enveloped cosine function (e.g., Chao et al., 2014). The noisiness of the pre-1992 data is evident. The dominant (retrograde) FCN signal across the wavelet spectrum at the (negative) period somewhat longer than a year is well captured. Note the fluctuations in the apparent period of this FCN signal, and the considerable time-undulations in amplitude which all but disappeared temporarily during the late 1990s. We shall return to these observations later.

3. Kinematics of FCN

It is crucial to consider two distinct fundamental reference frames. The reference frame set in the inertia space is referred to as $\boldsymbol{F_0}$, which is equivalent to the celestial reference frame to the best of its realization. The other reference frame, referred to as $\boldsymbol{F_\Omega}$, undergoes a uniformly rotation at a constant angular velocity $\boldsymbol{\Omega}$ w.r.t. the inertia space, $\boldsymbol{\Omega} = \Omega\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector pointing to the mean Earth rotation axis for the last half century, and the magnitude $\Omega = 2\pi$ radians per sd $= 7.292115 \times 10^{-5}$ rad s^{-1}, equivalent to 1/86 164 s^{-1} or 1 cycle per sd. $\boldsymbol{F_\Omega}$ is idealized in the sense that it is not observationally realized on the Earth but adequately approximates the diurnally rotating terrestrial reference frame to describe the physics below (e.g., Smith, 1977; Chao, 1983).

The transformation between $\boldsymbol{F_0}$ and $\boldsymbol{F_\Omega}$ is purely kinematic and rather simple: For a function $m(t) = m_x(t) + i m_y(t)$ describing a 2-D motion in the equatorial plain $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ with the Cartesian coordinates x (the real part) and y (the imaginary part), we can write, following Brzezinski and Capitaine (1993):

$$m'(t)[\text{w.r.t. } \boldsymbol{F_\Omega}] = m(t)[\text{w.r.t. } \boldsymbol{F_0}] \cdot \exp(-i\Omega t). \tag{1}$$

Here, contrary to the literature, we use the primed symbol to denote quantities w.r.t. $\boldsymbol{F_\Omega}$. Upon taking the Fourier transform, Equation (1) amounts to a simple shift in the angular frequency:

$$\omega'[\text{w.r.t. } \boldsymbol{F_\Omega}] = \omega[\text{w.r.t. } \boldsymbol{F_0}] - \Omega, \tag{2}$$

as depicted in Fig. 2. The positive frequency indicates prograde motion (as of CW), and negative frequency retrograde motion (as of FCN). In particular, the (retrograde) "nearly diurnal free wobble" (an old terminology for FCN) in $\boldsymbol{F_\Omega}$ transforms to the (retrograde) FCN with a near-zero frequency in $\boldsymbol{F_0}$ (thereof Equation (2) takes on a form $-1.003 = -0.003 - 1$ or thereabout when expressed in the frequency unit of cycle/sd). The periodicity seen in Fig. 1 (retrograde with negative period of somewhat longer than a year) corresponds to FCN's ω w.r.t. $\boldsymbol{F_0}$. For convenience we shall adopt the solar day as the time unit to conform to the VLBI observation unless specified otherwise, and convert the period to sd only in the end.

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