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A power spectrum for the geomagnetic dipole moment

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ABSTRACT

We use stochastic models for fluctuations in the dipole moment to derive a geomagnetic power spectrum. The theoretical spectrum is represented as a function of frequency f in the general form Af^{-n} with smooth transitions between changes in the exponent n. A flat spectrum ($n = 0$) at low frequencies changes to $n = 2$ at intermediate frequencies. We attribute the transition frequency to the decay time of dipole fluctuations. Accounting for correlated noise in the stochastic model introduces another transition from $n = 2$ to 4, where the transition frequency is set by the correlation time of the noise. Numerical geodynamo models suggest that the correlation time is less than the convective overturn time and may be related to the lifetime of helical eddies. Applying these results to paleomagnetic estimates of the power spectrum yields a correlation time of 100 to 200 yrs. Evidence for a transition between $n = 0$ and $n = 2$ in paleomagnetic power spectra is interpreted as a constraint on the electrical conductivity of the core. Additional correlation times in the noise model are expected to produce a transition to $n = 6$, possibly associated with diffusion of the dipole field through a magnetic boundary layer at the top of the core.

1. Introduction

Fluctuations in the geomagnetic field are observed over a broad range of time scales (Courtillot and LeMouël, 1988). Long-term trends in the mean reversal rate (Johnson et al., 1995) are observed with periods of 10^8 yrs or more. Such low-frequency phenomena are commonly attributed to slow convective processes in the Earth's mantle (e.g. Biggin et al., 2012). Dynamo processes are more naturally associated with higher frequency variability. A representative fluid velocity can sweep magnetic fields across the core in 10^2 yrs (Holme and Olsen, 2006), whereas the dipole decay time is roughly 10^4 yrs (Gubbins and Roberts, 1987). Waves with periods of 10^1 to 10^3 yrs can also contribute to the variability in the geomagnetic field (Gillet et al., 2010; Buffett, 2014; Canet et al., 2014).

A power spectrum for the geomagnetic field quantifies the variability and offers insights into the underlying processes (Constable and Johnson, 2005). An important source of information is obtained from measurements of relative paleointensity in marine sediments (Valet, 2003). Records are stacked and calibrated using independent estimates of absolute paleointensity to construct models for the virtual axial dipole moment (VADM) over the past two million years (Valet et al., 2005; Ziegler et al., 2011). Higher resolution records from a combination of archeomagnetic and lake sediment data have also been used to construct a low-degree spherical harmonic expansion of the geomagnetic field over the past 10 kyr (Korte et al., 2011; Korte and Constable, 2011). Even higher resolution records are available from historical observations (Jackson et al., 2000). Taken together these models provide a broadband estimate of fluctuations in the axial dipole moment.

A quantitative interpretation of the observed power spectrum requires physical models for the underlying processes. Numerical geodynamo models offer insights (Sakuraba and Hamano, 2007; Olson et al., 2012), but concerns about unrealistic model parameters invariably complicate a direct comparison with observations. We propose an alternative approach that uses stochastic models for the axial dipole moment to construct a theoretical power spectrum. The parameters of a stochastic model can be recovered from a realization of the process, so it is possible to use paleomagnetic estimates of the VADM to guide the construction of a power spectrum. The same procedures can also be applied to the output of geodynamo models. By comparing the theoretical power spectrum from the stochastic model with that computed directly from the geodynamo model, we build confidence in the approach and attach physical significance to features in the resulting power spectrum. Extending these ideas to paleomagnetic estimates of the VADM provides a physical basis for interpreting the observed power spectrum.

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2. Stochastic description of dipole fluctuations

Stochastic models are often applied to problems where only a small subset of variables are observable. The approach is well suited to study of the geomagnetic dipole moment because most of the convective flow that sustains the dipole field is not detected at the surface. A stochastic model separates the evolution of the dipole field into two parts, based on the inherent time scales of the processes involved. One part describes the slow adjustment of the dipole field toward a time-averaged state, while the second part represents the effect of short-period convective fluctuations in the core. These short-period fluctuations are treated as a random process in the time evolution. We denote the axial dipole moment by $x(t)$ and describe its time evolution using a standard Langevin model (Van Kampen, 2007)

$$\frac{dx}{dt} = v(x) + \sqrt{D(x)}\Gamma(t) \tag{1}$$

where the drift term, $v(x)$, describes the slow evolution and the noise term, $D(x)$, defines the amplitude of random fluctuations. The time dependence of the random process, $\Gamma(t)$, is assumed to be Gaussian with a vanishing time average

$$\big\langle \Gamma(t) \big\rangle = 0. \tag{2}$$

We also assume that the correlation time of the noise source is short compared with the sampling of $x(t)$, although this assumption is relaxed in later sections. In the limit of short correlation times the autocovariance function of $\Gamma(t)$ is approximated by a Dirac delta function,

$$\big\langle \Gamma(t_1)\Gamma(t_2) \big\rangle = 2\delta(t_1 - t_2) \tag{3}$$

where the factor of two is a common convention (e.g. Risken, 1989).

Estimates for $v(x)$ and $D(x)$ can be extracted from a realization of the stochastic process. The drift term is given by

$$v(x) = \frac{\langle x(t+\Delta t) - x(t) \rangle}{\Delta t} \tag{4}$$

and the noise term is approximated by

$$D(x) = \frac{\langle [x(t+\Delta t) - x(t)]^2 \rangle}{2\Delta t} \tag{5}$$

where time averages are assigned according to the position of $x(t)$ in discrete bins that span the range of dipole moments (see Buffett et al., 2014 for details). The time increment, Δt, is chosen to be long enough to ensure that the noise source, $\Gamma(t)$, is adequately approximated by an uncorrelated random process. Fig. 1 shows an example using the output of a numerical geodynamo model. The drift term can be represented by

$$v(x) = -\gamma\big(x - \langle x \rangle\big) \tag{6}$$

where $\langle x \rangle$ denotes the time-averaged moment and γ is a constant that characterizes the time scale for slow adjustments of the dipole (Buffett et al., 2014). A similar representation for $v(x)$ was recovered from paleomagnetic estimates (e.g. Buffett et al., 2013). Indeed very similar values for the constant, $\gamma \approx 34$ Myr^{-1}, were reported for the SINT-2000 model of Valet et al. (2005) and the PADM2M model of Ziegler et al. (2011). By comparison, the noise term in Fig. 1 reveals a weak dependence on x. It suffices for our purposes to adopt the approximation $D(x) = D_{eq}$, where D_{eq} denotes the value of the noise term at $x = \langle x \rangle$. We explore the validity of this approximation below.

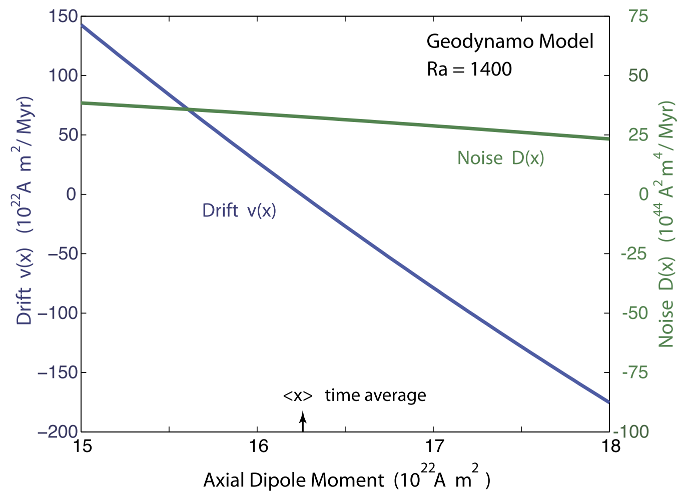

Fig. 1. Drift and noise terms as a function of dipole moment x from the geodynamo model *Calypso* (see text). The drift term is well approximated by a linear function of x, whereas the noise term is nearly constant.

2.1. Power spectrum for the dipole moment

A power spectrum is computed for deviations $\epsilon(t) = x(t) - \langle x \rangle$ around the time average. The equation for $\epsilon(t)$ becomes

$$\frac{d\epsilon}{dt} = -\gamma\epsilon + \sqrt{D_{eq}}\Gamma(t) \tag{7}$$

and solutions are obtained using Fourier transforms. We define the Fourier transform of $\epsilon(t)$ in terms of frequency, f, as

$$\epsilon(f) = \int_{-\infty}^{\infty} \epsilon(t) e^{-2\pi i f t}\, dt \tag{8}$$

An analogous expression defines $\Gamma(f)$. Taking the Fourier transform of (7) yields

$$\epsilon(f) = \frac{\sqrt{D_{eq}}}{(\gamma + i2\pi f)}\Gamma(f) \tag{9}$$

Consequently, the power spectrum of the dipole moment becomes

$$S_\epsilon(f) \equiv \epsilon(f)\epsilon(f)^* = \frac{D_{eq}}{(\gamma^2 + 4\pi^2 f^2)} S_\Gamma(f) \tag{10}$$

where the power spectrum of the noise

$$S_\Gamma(f) \equiv \Gamma(f)\Gamma(f)^* = 2 \tag{11}$$

is obtained by taking the Fourier transform of the autocovariance function for $\Gamma(t)$ in (3). Thus the predicted spectrum in (10) is nearly constant at low frequency and decreases at higher frequency as f^{-2}. A qualitatively similar power spectrum was proposed by Barton (1982) on the basis of paleomagnetic directions. More quantitative agreement is found with the study of Olson et al. (2012), which analyzed the dipole moment from a suite of numerical geodynamo models. The typical spectrum in their study was flat at low frequency and decreased as $f^{-1.8\pm0.1}$ at higher frequencies. Similar results were obtained in the study of Davies and Constable (2014), which recovered a frequency exponent of $n = 2.1 \pm 0.2$.

On the basis of the stochastic model, the transition in the power spectrum is approximated by $f \approx \gamma/2\pi$, where γ^{-1} represents the time scale for slow adjustments in the dipole moment.