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The transition to Earth-like torsional oscillations in magnetoconvection simulations

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ABSTRACT

Evidence for torsional oscillations (TOs) operating within the Earth's fluid outer core has been found in the secular variation of the geomagnetic field. These waves arise via disturbances to the predominant (magnetostrophic) force balance believed to exist in the core. The coupling of the core and mantle allow TOs to affect the length-of-day of the Earth via angular momentum conservation.

Encouraged by previous work, where we were able to observe TOs in geodynamo simulations, we perform 3-D magnetoconvection simulations in a spherical shell in order to reach more Earth-like parameter regimes that proved hitherto elusive.

At large Ekman numbers we find that TOs can be present but are typically only a small fraction of the overall dynamics and are often driven by Reynolds forcing at various locations throughout the domain. However, as the Ekman number is reduced to more Earth-like values, TOs become more apparent and can make up the dominant portion of the short timescale flow. This coincides with a transition to regimes where excitation is found only at the tangent cylinder, is delivered by the Lorentz force and gives rise to a periodic Earth-like wave pattern, approximately operating on a 4 to 5 year timescale. The core travel times of our waves also become independent of rotation at low Ekman number with many converging to Earth-like values of around 4 years.

1. Introduction

The geodynamo, which operates in the Earth's iron-rich outer core and continuously replenishes its magnetic field, is believed to be one of a number of planetary dynamos that exists under a quasi-magnetostrophic regime (see, for example, Jones et al., 2011). Specifically, this suggests that, owing to the rapidly rotating nature of the Earth, the predominant force balance within the core is between Lorentz, Archimedean and Coriolis forces; this is commonly known as the MAC balance. One consequence of the magnetostrophic regime is that the averaged azimuthal Lorentz force is compelled to vanish on cylinders aligned with the rotation axis, leaving the system in a Taylor state (Taylor, 1963). Violations of Taylor's constraint manifest themselves as the acceleration of concentric cylinders with a restoring Lorentz force acting like a torsional spring that attempts to reimpose the Taylor state (Braginsky, 1970). This process ultimately leads to the excitation of torsional oscillations (TOs), which are the only Alfvén waves (Alfvén, 1942) in the core that act at large lengthscales, propagating in the cylindrical radial direction.

Theory has been complemented, in recent years, by improved observational data although there have been conflicting estimates of the timescale on which TOs operate. Braginsky (1984) first suggested a 60-year timescale, however, this has now been superseded by recent core flow models (Gillet et al., 2010) as well as signals in length-of-day data (Holme and de Viron, 2013; Chao et al., 2014), which indicate a 6 year operational timescale. Indeed, TOs have long been presented as an explanation for certain changes in the Earth's length-of-day via the coupling of solid and fluid regions at the core–mantle boundary (CMB) by angular momentum conservation (Jault et al., 1988; Jackson, 1997; Roberts and Aurnou, 2012), and may be connected with other features such as geomagnetic jerks (Bloxham et al., 2002; Brown et al., 2013).

Direct observation of TOs is inhibited by the inability to probe the Earth's magnetic interior beneath the CMB (Gubbins and Bloxham, 1985), though it is believed that the TO velocity field can be seen in the recent secular variation data (Gillet et al., 2010). However, further theoretical progress can be made; in particular, improved computing resources in recent years has allowed for the

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identification of TOs in numerical simulations (Dumberry and Bloxham, 2003; Busse and Simitev, 2005; Wicht and Christensen, 2010; Teed et al., 2014). Wicht and Christensen (2010) provided evidence of TOs operating in the region outside the tangent cylinder (OTC) and in a recent study we (from this point on referred to as TJT, Teed et al., 2014) extended this to include the region inside the tangent cylinder (ITC) where we observed waves crossing the tangent cylinder (TC). The exact excitation mechanism of TOs, both in simulations and the Earth itself, remains unclear but progress has been made in identifying the forcing terms involved (Wicht and Christensen, 2010), which are directly calculable in numerical models. Indeed the Lorentz and Reynolds forces are shown (TJT) to be important in simulations and it is likely that they can be in Earth's core also, although other excitation mechanisms are possible (Mound and Buffett, 2006; Légaut, 2005).

As is the case with all numerical models of the Earth's core, the search for TOs is currently hindered by our inability to reach the actual parameter values of the core. The ratio of viscous to Coriolis forces is tiny in the core and it is not possible to get close to the quasi-magnetostrophic regime for dynamo calculations. Having reached the numerical limits of our code in TJT, a new approach has been necessary to continue our investigation of TOs in simulations and this is the topic of the work presented here. Very long runs are needed to establish the growth and equilibration of a dynamo driven magnetic field. Since we are now confident that numerical dynamos giving a field similar to the geomagnetic field exist in the appropriate region of parameter space, we can make significant computational savings by imposing the dipole component of the magnetic field at the CMB. The transient field state is then much shorter, and this allows us to reach more Earth-like parameters in this work. Also, magnetoconvection simulations provide the additional benefit of allowing us to vary the imposed magnetic field strength as an input parameter to allow for a systematic exploration of the dependence of TOs on field strength.

We identify a condition for the observation of TOs in the presence of overlying convection in simulations, based on an output parameter, the short time-scale geostrophy parameter, introduced in TJT. We explore the driving forces, based on the novel approach used in TJT of separating the Lorentz force into its restoring and driving components, as well as the driving locations of the waves as we move into new parameter regimes inaccessible to us hitherto. Core travel times based on the Alfvén speed are also calculated.

2. Mathematical formulation and methods

2.1. Mathematical model

The model we employ is adapted for magnetoconvection from that used in TJT where dynamo runs were performed. Hence, using a spherical coordinate system (r,θ,ϕ), we consider a fluid filled spherical shell that rotates about the vertical with rotation rate Ω ($\mathbf{\Omega}=\Omega\hat{\mathbf{z}}$). Gravity acts radially inward ($\mathbf{g}=-g\mathbf{r}$) and the fluid is modelled using the Boussinesq approximation with constant values of ρ, ν, κ and η, the outer core density, kinematic viscosity, thermal diffusivity and magnetic diffusivity respectively.

The geomagnetic field is frequently decomposed into spherical harmonics, and in our numerical code the magnetic field is separated into poloidal and toroidal parts with each part being split into spherical harmonics (see, for example, Roberts, 2007). At an insulating boundary there is a matching condition between each poloidal component and its normal derivative so it is therefore convenient to implement magnetoconvection by setting the amplitude of the axial dipole component, Y_1^0, so that this component gives $\mathbf{B_0}=(2B_0\cos\theta, B_0\sin\theta, 0)$ on the CMB at $r=r_o$. On all the remaining components of the field, both at $r=r_o$ and at $r=r_i$, we use the standard form of insulating magnetic boundary conditions (for further details see the Supplementary Material).

We previously (TJT) implemented fixed flux thermal boundary conditions; this was to promote Earth-like dynamo regimes, which, under magnetoconvection, is no longer a requirement. Therefore, in a change from TJT, we return to the simpler choice of fixed temperature conditions so that convection is instigated by differential heating. We also impose rigid (no-slip) kinematic boundary conditions at both boundaries.

The system of coupled equations for velocity, $\mathbf{u}$, magnetic field, $\mathbf{B}$, temperature, T, and pressure, p are:

$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=-\frac{Pm}{E}\left[\nabla p+2\hat{\mathbf{z}}\times\mathbf{u}-(\nabla\times\mathbf{B})\times\mathbf{B}\right]+\frac{Pm^2Ra}{Pr}T\mathbf{r}+Pm\nabla^2\mathbf{u}, \tag{1}$$

$$\frac{\partial T}{\partial t}+(\mathbf{u}\cdot\nabla)T=\frac{Pm}{Pr}\nabla^2T, \tag{2}$$

$$\frac{\partial\mathbf{B}}{\partial t}-\nabla\times(\mathbf{u}\times\mathbf{B})=\nabla^2\mathbf{B}, \tag{3}$$

$$\nabla\cdot\mathbf{u}=0, \tag{4}$$

$$\nabla\cdot\mathbf{B}=0, \tag{5}$$

where we have nondimensionalised using length scale, $D=r_o-r_i$, magnetic timescale, D^2/η, temperature scale, ΔT, and magnetic scale, $\sqrt{\rho\mu_0\Omega\eta}$.

The nondimensional parameters appearing in our equations are the Rayleigh number, Ra, Ekman number, E, Prandtl number, Pr, and magnetic Prandtl number, Pm, defined by:

$$Ra=\frac{g\alpha\Delta T D^3}{\nu\kappa},\quad E=\frac{\nu}{\Omega D^2},\quad Pr=\frac{\nu}{\kappa},\quad Pm=\frac{\nu}{\eta}. \tag{6}$$

We take the value of the aspect ratio of the spherical shell, given by r_i/r_o, to be 0.35 throughout, which is apt for a model of the Earth's core.

2.2. TO theory

The observation of TOs requires the definition of various spatial and temporal averages in a cylindrical coordinate system: (s,ϕ,z). We continue the notation from TJT so, for any scalar field, $A(t,s,\phi,z)$, we define

$$\bar{A}(t,s,z)=\frac{1}{2\pi}\int_0^{2\pi}A\,d\phi\quad\text{and}$$

$$\langle A\rangle(t,s,\phi)=\frac{1}{h}\left(\int_{-z_+}^{-z_-}A\,dz+\int_{z_-}^{z_+}A\,dz\right), \tag{7}$$

for spatial averages over ϕ and z, where $z_+=\sqrt{r_o^2-s^2}$, $z_-=\Re\left\{\sqrt{r_i^2-s^2}\right\}$, $h=2(z_+-z_-)$. A complication arose at this stage in TJT, where multiple definitions of the z-average were required to account for the separation of hemispheres by the inner core and the inhomogeneous nature of the fields in the dynamo runs, when $s<r_i$. Although the current geometry maintains this separation, we find that in practice the imposition of a dipolar magnetic field on the outer boundary creates a restriction so that the dynamics in the two hemispheres are very similar. The requirement for different definitions of the z-average OTC and ITC is therefore lifted and our z-averages in this work are calculated across the entire domain for all values of s. In TJT we found it helpful to introduce