Earth and Planetary Science Letters 403 (2014) 117–130

Contents lists available at ScienceDirect

Earth and Planetary Science Letters

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A model for eruption frequency of upper crustal silicic magma chambers

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A R T I C L E   I N F O

Article history:
Received 24 January 2014
Received in revised form 23 June 2014
Accepted 27 June 2014
Available online 15 July 2014
Editor: T. Elliott

Keywords:
magma chamber
eruption frequency
eruption trigger

A B S T R A C T

Whether a magma body is able to produce eruptions and at what frequency remains a challenging problem in volcanology as it involves the nonlinear interplay of different processes acting over different time scales. Due to their complexity these are often considered independently in spite of their coupled nature. Here we consider an idealized model that focuses on the evolution of the thermodynamic state of the chamber (pressure, temperature, gas and crystal content) as new magma is injected into the chamber. The magma chamber cools in contact with the crust, which responds viscoelastically to the pressure accumulated during recharge and volatile exsolution. The magma is considered eruptible if the crystal volume fraction is smaller than 0.5. If a critical overpressure is reached, mass is released from the magma chamber until the lithostatic pressure is recovered. The setup of the model allows for rapid calculations that provide the opportunity to test the influence of competing processes on the evolution of the magma reservoir. We show how the frequency of eruptions depends on the timescale of injection, cooling, and viscous relaxation and develop a scaling law that relates these timescales to the eruption frequency. Based on these timescales we place different eruption triggering mechanisms (second boiling, mass injection, and buoyancy) in a coherent framework and evaluate the conditions needed to grow large magma reservoirs.

1. Introduction

Explosive eruption frequency of volcanoes is inversely correlated with the associated erupted volume (Newhall and Self, 1982). Explosive eruptions produce volumes that range from $<10^{-2}$ km^3 being erupted nearly continuously (Houghton et al., 2013) to $>10^3$ km^3 occurring less than once every 100,000 years (Mason et al., 2004; Deligne et al., 2010). This is due to the way magma ascends into crust, forms a volcanic plumbing system, and interacts with its surroundings (Scandone et al., 2007). Small and frequent eruptions are governed by the conduit system that transports magma to the Earth's surface (e.g., Voight et al., 1999; Burton et al., 2007), while the larger explosive eruptions are believed to be controlled by the magma reservoir dynamics in the upper crust (Jellinek and DePaolo, 2003; Caricchi et al., 2014; Malfait et al., 2014). In particular, the evolution of a shallow magma chamber will have a dominant impact on the long-term eruption history of a volcano. To understand the nature of this impact the nonlinear interplay between the mechanisms governing the magma chamber's evolution has been studied using mathematical models (e.g., Marsh, 1981). Numerous models have been developed for understanding both the compositional and dynamical evolution of magma chambers (e.g., Mourtada-Bonnefoi et al., 1999; Tait et al., 1989; Jellinek and DePaolo, 2003; Spera and Bohrson, 2004; Michaut and Jaupart, 2006; Fowler and Spera, 2008, 2010; Huber et al., 2009, 2010a, 2010b, 2011; Dufek and Bachmann, 2010; Karlstrom et al., 2010).

Magma chambers will produce an eruption if they can initiate and propagate a dyke that reaches the Earth's surface. In this study we define an eruption to be the initiation of a dyke and do not consider whether it will reach the Earth's surface (Taisne et al., 2011). A dyke can be nucleated by a chamber if it (i) contains mobile magma (e.g., Marsh, 1981) and (ii) produces sufficient overpressure (e.g., Rubin, 1995). In order to be mobile the magma needs to contain enough enthalpy so as to remain above a critical crystal fraction where it becomes mechanically locked (Marsh, 1981; Lejeune and Richet, 1995; Caricchi et al., 2007; Champallier et al., 2008). This behavior is controlled by heat inflow provided from deeper sources, the chambers ability to flux heat out to the colder surrounding crust and the phase diagram of the magma. Overpressure can be generated by mass inflow (Blake, 1981; Jellinek and DePaolo, 2003), crystallization-induced exsolution ('second boiling'; Blake, 1984; Tait et al., 1989; Folch and Marti, 1998; Snyder, 2000; Fowler and Spera, 2008, 2010),

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http://dx.doi.org/10.1016/j.epsl.2014.06.047
0012-821X/

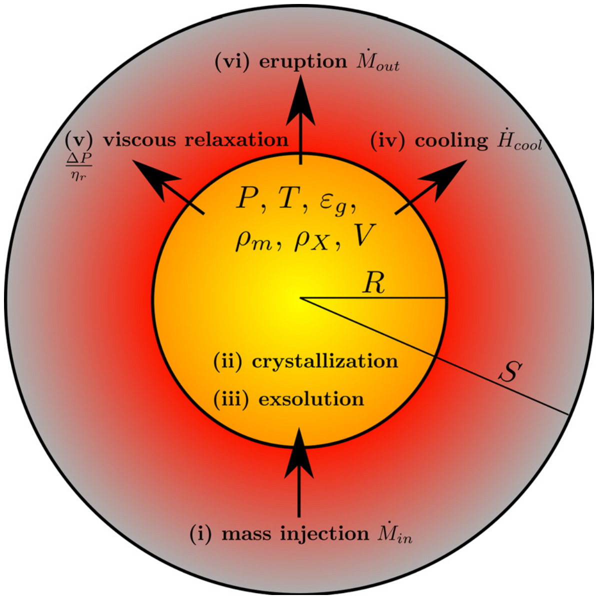

Fig. 1. Schematic representation of the magma chamber box model. A constant rate of mass is injected into a spherical chamber of radius R with a homogeneous magma mixture while it is cooled by a colder, visco-elastic crustal shell of radius S. Crystallization and exsolution occur within the chamber changing the relative abundances of the melt, crystal and gas phases. When a critical overpressure is reached before mechanical locking an eruption can occur. We track the evolution of the canonical variables pressure P, temperature T, and gas volume fraction ε_g, melt density ρ_m, crystal density ρ_X, and chamber volume V. All other quantities are derived from these.

and buoyancy (Jellinek and DePaolo, 2003; Caricchi et al., 2014; Malfait et al., 2014). External factors such as near-field seismicity (Gottsmann et al., 2009), roof failure (Gregg et al., 2012), and tectonic extension (e.g., Catalano et al., 2014) can also affect the ability to initiate and propagate a dyke from the magma chamber. The excess pressure from mass inflow and second boiling in the reservoir can be erased by an eruption, viscous relaxation of the crust or passive degassing. These essential ingredients for an eruption have been modeled separately, but need to be considered simultaneously to evaluate a chambers ability to erupt (Karlstrom et al., 2010).

We focus on the evolution of upper crustal silicic magma chambers and the amount of eruptions they can potentially produce before becoming mechanically locked. Jellinek and DePaolo (2003) have shown that in order for magma to accumulate and produce large-volume chambers the timescale of magma injection has to be greater than the timescale to relax the overpressure by crustal deformation. However, the timescale of injection has to be smaller than the cooling timescale in order to sustain enough heat and maintain a mobile magma batch within the crust (Annen, 2009; Gelman et al., 2013). Building a large chamber thus requires an inflow rate high enough to keep the magma mobile, yet low enough to allow for mass accumulation by viscous relaxation of the crust. We develop a numerical model that considers all these timescales through simplified parameterizations. This allows us to simulate the coupling between the processes directly and to rapidly evaluate a large parameter space of different initial conditions. We analyze eruption occurrence resulting from the model as a function of the timescales involved and derive relationships between the eruption frequency, the magma chamber radius and the mass inflow rate.

2. Methods

We develop a box model to study the coupled processes controlling magma chamber evolution (Fig. 1). The magma chamber is considered to be a homogeneous sphere of volume V and is assumed to remain spherical at all time. This assumption allows us to describe the chamber by a single pressure P, temperature T, and volume fraction of melt ε_m, gas ε_g, and crystals ε_X and to develop analytical expressions for the pressure buildup in the chamber and the temperature field around the chamber as a function of time. The melt, gas, and crystal phase are assumed to be in thermodynamic equilibrium. We consider the processes of (i) mass injection (ii) crystallization, (iii) exsolution, (iv) cooling (v) viscous relaxation, and (vi) eruption. We assume mass is injected into the chamber at a constant rate. The magma chamber sits in a colder spherical crustal shell, which reacts visco-elastically to pressure changes in the chamber. The chamber can lose mass through an eruption if the pressure reaches a critical value before the crystal volume fraction is too high (here 0.5). We calculate the evolution of the independent variables P, T, and ε_g over time by solving (i) the conservation of total mass, (ii) the conservation of water, and (iii) the conservation of total enthalpy. We further introduce closure equations for the evolution of the melt density ρ_m, the crystal density ρ_X, and the chamber volume V, based on melt, crystal, and crustal properties, respectively. All other quantities can then be calculated as a function of the canonical variables P, T, ε_g, ρ_m, ρ_X, and V. The calculation of the magma chamber evolution is stopped when the crystal volume fraction reaches 0.5. Parameterizations for the equation of state of the gas phase, the melting curve, the solubility curve, the temperature field in the crust, and the viscosity of the crust are described in Appendix A. The symbols used are defined in Table 1.

2.1. Conservation of total mass

Our mathematical description of the model begins by considering the conservation of (total) mass,

$$\frac{dM(P, T, \varepsilon_g, \rho_m, \rho_X, V)}{dt} = \dot{M}_{\text{in}} - \dot{M}_{\text{out}}, \tag{1}$$

with t time, M the total mass, $\dot{M}_{\text{in}}$ the mass inflow rate, and $\dot{M}_{\text{out}}$ the mass outflow rate. The mass inflow rate is considered to be a constant. The mass outflow rate is initially set to zero. Rheology experiments have shown a drastic decrease in magma mobility at a crystal volume fraction between 0.4 and 0.6 (Lejeune and Richet, 1995; Caricchi et al., 2007; Champallier et al., 2008) and Rubin (1995) reports dike growth can become critical at excess pressures between 10 MPa and 50 MPa. We therefore consider an eruption to occur when the crystal volume fraction is below 0.5 and a critical overpressure of $(\Delta P)_c = 20$ MPa is reached. Once these conditions are met, the mass outflow rate is set to $\dot{M}_{\text{out}} = 10^4$ kg s^{-1}. This is much faster than any other process considered in the chamber while being representative of published estimates of eruption rates (Pyle, 2000). Once the pressure returns to lithostatic, the mass outinflow rate is reset to zero.

We wish to write the conservation of mass in its canonical form, i.e. as a function of the time derivatives of the independent variables P, T, and ε_g. To do so we will introduce the constitutive equations that we need as we proceed. Starting from the definition of total mass,

$$M(P, T, \varepsilon_g, \rho_m, \rho_X, V) = \rho(P, T, \varepsilon_g, \rho_m, \rho_X)V \tag{2}$$

with ρ the mixture (melt + crystals + gas) density and V the volume of the magma chamber, we can rewrite Eq. (1) as

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{V}\frac{dV}{dt} = \frac{\dot{M}_{\text{in}} - \dot{M}_{\text{out}}}{\rho V}. \tag{3}$$

The mixture density is defined as