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Theoretical analysis of particle number density in steady aeolian saltation

Liqiang Kang *, Xueyong Zou

State Key Laboratory of Earth Surface Processes and Resource Ecology, Beijing Normal University, Beijing 100875, China
MOE Engineering Research Center of Desertification and Blown-sand Control, Beijing Normal University, Beijing 100875, China

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ABSTRACT

Particle number density or particle concentration in aeolian saltation is one important input parameter to calculate the sand flux, kinetic energy and mid-air collision probability in the aeolian saltation and particle concentration is also related to the wind erosion capacity, hence, in the present paper, the vertical distribution of particle number density in steady aeolian saltation is analyzed based on two different types of probability density functions of vertical lift-off velocity of saltating particles: one is the PDF (probability density function) of vertical velocity of lift-off particles in the three-dimensional space defined as a type-A PDF which considers the number of particles in various velocity bins per unit volume; and the other is the PDF of vertical velocity of lift-off particles ejected from the sand bed surface in a period of time as a type-B PDF which considers the number flux of particles in various velocity bins per unit surface area. These two types of PDFs are from two different perspectives (i.e., volume- and surface-based perspectives, respectively), and can be deduced from each other. The half-normal and exponential distributions are recommended for the type-A PDF, and the corresponding type-B PDF is expressed by Rayleigh and Gamma(2) distributions. The PDF distribution pattern of vertical velocity of lift-off particles has an important influence on the vertical profile of particle number density. If the type-A PDF of vertical velocity of ejected particles is a half-normal distribution, the particle number density decays exponentially with height. If the type-A PDF is an exponential distribution, the particle number density also decreases with height. If the type-A PDF is Gamma(3) and Rayleigh distributions, the particle number density first increases, then decreases with height. The type-A and type-B height parameters, which are calculated according to the mean vertical lift-off velocity from the type-A and type-B PDFs, respectively, are not simply considered as the actual mean saltation height, although if the type-A PDF of vertical lift-off velocity is half-normal PDF, the mean dispersion height of particle concentration equals the mean saltation height. The vertical distribution of particle number density can determine the dislodgement rate on bed surface which further affects the surface bedforms.

1. Introduction

Sand particles on the bed surface can be driven into the air under strong winds, then windblown sand movement occurs, in which saltation is the dominant mode of blown sand movement accounting for about 75% of the total sand flux (Bagnold, 1941). This phenomenon exists widely in natural environments and is an important geomorphological process for landform change on the Earth and other planets such as Mars, Venus and Titan (Greeley and Iversen, 1985).

Particle number density or particle concentration in aeolian saltation is one important input parameter to calculate the sand flux and kinetic energy of a saltating cloud, and then the vertical distribution of particle number density or particle concentration is also related to the bed erosion and the formation of aeolian bedforms (from small scale ripples to the larger scale dunes) caused by the saltating grains. Generally, if the sand flux increases (or the erosion intensity increases), the particle concentration also increases (e.g., Liu and Dong, 2004; Creyssels et al., 2009). In the model of Sørensen and McEwan (1996), the vertical profile of particle concentration is also an input parameter to estimate the mid-air collision probability. The mid-air collisions can modify the grain behavior and reduce the number of grain-bed impacts, and then further influence the sand transport rate (or the erosion intensity). The variation of particle concentration with height also reflects the probability distribution of the vertical ejected velocity of the saltating particles (see Section 3 of this paper), and conversely the probability distribution of the vertical lift-off velocity is probably derived from the vertical profile of particle concentration. As is well known, the lift-off velocity probability distribution is a bridge for linking microscopic and macroscopic aeolian research, and is helpful for estimating the statistical parameters of saltating grains. Hence, the particle number density or particle concentration is highly relevant to the study of aeolian geomorphology.

In general, there are two types of approaches to study the particle concentration profile in windblown sand movement: wind tunnel experiments (Dong et al., 2003; Liu and Dong, 2004; Wang et al., 2006; Creyssels et al., 2009) and theoretical models including analytical

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* Corresponding author at: State Key Laboratory of Earth Surface Processes and Resource Ecology, Beijing Normal University, Beijing 100875, China. Tel./fax: +86 10 62207162.
E-mail address: kangliqiang@bnu.edu.cn (L. Kang).

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and numerical models (Nalpanis et al., 1993; Sørensen and McEwan, 1996; Jenkins et al., 2010; Kang, 2012). The reliable theoretical model is very important to predict the particle concentration in blowing sand transport. In the computed model of Nalpanis et al. (1993), particle concentration is calculated by counting the number of particles in a given region of space at certain times, and the result shows the concentration decays exponentially with height. With the saltation model of McEwan and Willetts (1991), Sørensen and McEwan (1996) gave that the calculated particle concentration decreases exponentially with height. However, the non-exponential profile of particle concentration is also reported (Jenkins et al., 2010; Kang, 2012). Jenkins et al. (2010) proposed a continuum model for steady saltation above a horizontal sand bed, and found that when turbulent suspension is considered, the particle concentration deviates from the exponential profile, especially near the sand bed region. In a discrete particle model, Kang (2012) obtained that the simulated particle volume concentration decreases exponentially with height at higher height and deviates from the exponential profile near the sand bed region. Therefore, a theoretical model study of particle concentration profile is still needed.

The particle-bed collision process or the ‘splash’ process becomes an important sub-process in some models (e.g., Ungar and Haff, 1987; Anderson and Haff, 1988, 1991; McEwan and Willetts, 1991) by inputting the initial lift-off velocity distribution of saltating grains into these numerical models. The ejected velocity distribution of saltating grains further affects the saltation trajectories and the statistical parameters such as particle number density and mass flux. However, because of the complexity of particle-bed collisions, the reported resultant lift-off velocity distributions of saltating particles have many types of distribution patterns such as exponential function, Weibull function, log-normal and Gamma distributions (Anderson and Haff, 1988, 1991; Nalpanis et al., 1993; Dong et al., 2002; Cheng et al, 2006; Kang et al., 2008). For the vertical lift-off velocity distributions of saltating particles, the reported distribution patterns can be described as exponential and Gamma distribution functions (Anderson and Hallet, 1986; Cheng et al, 2006; Kang et al., 2008). Hence, in order to provide the more reasonable lift-off velocity distribution function for the theoretical model, it is necessary to further study the distribution pattern of lift-off velocity of saltating particles.

In this paper, two different types of probability density functions of vertical lift-off velocity of saltating particles are analyzed and defined as type-A and type-B PDFs, respectively. Then based on these two types of PDFs, the vertical distribution of particle number density in steady saltation is deduced. According to the relation of the two types of PDFs, the possible distribution patterns are determined for type-A and type-B PDFs of vertical lift-off velocity of saltating particles. Finally, the results and some characteristic parameters are discussed.

2. Two different probability density functions of vertical lift-off velocity

There are two different types of probability density functions (PDF) of vertical lift-off velocity of saltating particles: one is the PDF of vertical velocity of lift-off particles obtained from the statistics of lift-off particles in the three-dimensional volume close to the sand bed surface at a given time, for simplicity, we define this as a type-A PDF, shown in Fig. 1(a); and the other is the PDF of vertical velocity of lift-off particles obtained by analyzing the lift-off particles ejected from the sand bed surface in a period of time per unit surface area, defined here as a type-B PDF, shown in Fig. 1(b).

The type-A PDF treatment involves a quantity that considers the number of particles (in various velocity bins) per unit volume whereas the type-B PDF treatment considers the number flux of particles (in various velocity bins) per unit surface area. The two types of PDFs are considered from two different perspectives, i.e., volume- and surface-based perspectives.

We consider that v_0 is the vertical lift-off velocity of saltating particles. $P_A(v_0)$ and $P_B(v_0)$ are the type-A and type-B PDFs of vertical velocity of lift-off particles, respectively. dv_0 is the interval width for vertical lift-off velocity, and $v_{0,i}$ is the characteristic velocity in the statistical interval $\left[v_{0,i}-\frac{dv_0}{2},\ v_{0,i}+\frac{dv_0}{2}\right]$, where the subscript i means the ith vertical lift-off velocity bin. Hence, the type-A probability density $P_A(v_{0,i})$ of lift-off vertical velocity can be expressed by:

$$P_A\left(v_{0,i}\right)=\frac{N_{A0,i}}{N_{A0}dv_0} \tag{1}$$

where $N_{A0,i}$ is the number of lift-off particles with vertical velocity between $v_{0,i}-\frac{dv_0}{2}$ and $v_{0,i}+\frac{dv_0}{2}$ in the statistical three-dimensional volume closed to the sand bed surface. N_{A0} is the total number of lift-off particles in this statistical three-dimensional volume, $N_{A0}=\sum_{i=1}^{k}N_{A0,i}$, where k is the total number of statistical velocity intervals.

Letting $N_{B0,i}$ be the number of lift-off particles within vertical velocity interval $\left[v_{0,i}-\frac{dv_0}{2},v_{0,i}+\frac{dv_0}{2}\right]$ ejected from the sand bed surface per unit time per unit area, then

$$N_{B0,i}=\frac{N_{A0,i}}{dV}v_{0,i} \tag{2}$$

where dV is the statistical three-dimensional volume, shown in Fig. 1(a).

The total number of lift-off particles leaving the sand bed surface per unit time per unit area, i.e., the number flux of sand particles leaving the sand bed surface, is

$$N_{B0}=\sum_{i=1}^{k}N_{B0,i}=\sum_{i=1}^{k}\frac{N_{A0,i}v_{0,i}}{dV}. \tag{3}$$

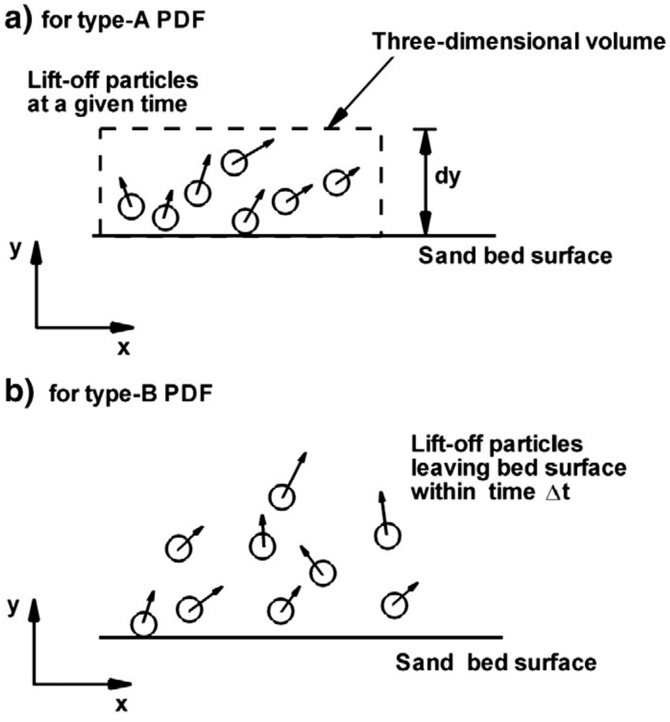

Fig. 1. Sketch to explain the considered particles of the type-A and type-B PDFs of vertical lift-off velocity.