Tectonophysics 608 (2013) 1371–1379

Contents lists available at ScienceDirect

Tectonophysics

journal homepage: www.elsevier.com/locate/tecto

Dynamical similarity and density (non-) proportionality in experimental tectonics

N.M. Ribe *, A. Davaille

Lab FAST, Université Paris-Sud and CNRS, Bât. 502, Campus Universitaire, 91405 Orsay, France

ARTICLE INFO

Article history:
Received 8 March 2013
Received in revised form 27 May 2013
Accepted 4 June 2013
Available online 13 June 2013

Keywords:
Experimental tectonics
Dynamical similarity
Scale models
Subduction

ABSTRACT

The utility of analog laboratory models for tectonic processes relies on their dynamical similarity to their natural prototypes. Dynamical similarity is often thought to require that the density distribution in the model be a constant (position-independent) multiple of that in the prototype, a principle due to Hubbert (1937). To clarify the status of this rule, we nondimensionalize the equations and boundary/initial conditions governing simple models of three paradigmatic processes: gravity tectonics, compressional tectonics, and free subduction. The results show that density proportionality, while compatible with dynamical similarity, is not always required by it in systems with negligible inertia, a category that includes most geological and tectonic processes. The density proportionality rule is therefore unnecessarily restrictive in many cases, implying that the range of analog materials that can be used to construct properly scaled models is wider than commonly recognized.

1. Introduction

Analog laboratory models are a versatile and powerful tool for understanding the origin of complex geological and tectonic structures. Their use has a long history, beginning with Sir James Hall's (1815) experiments on folding under compression. Since Hall's time many hundreds of analog modeling studies of diverse tectonic phenomena have appeared in the literature. Examples from recent decades include investigations of folding and boudinage (Abbassi and Mancktelow, 1992; Biot et al., 1961; Cobbold, 1975; Hudleston, 1973; Mengong and Zulauf, 2006; Neurath and Smith, 1982; Ramberg, 1959, 1962; Treagus, 1972), diapirs and salt domes (Brun and Fort, 2004; Del Ventisette et al., 2005; Ramberg, 1967; Talbot et al., 1991; Weijermars et al., 1993), orogenic wedges (Cowan and Silling, 1978; Davis et al., 1983; Graveleau et al., 2012; Hubbert, 1951; Liu and Dixon, 1991; Lohrmann et al., 2003; Mulugeta, 1988), large-scale deformation of continental lithosphere (Allemand and Brun, 1991; Brun and Beslier, 1996; Davy and Cobbold, 1988, 1991; Tapponnier et al., 1982; Tron and Brun, 1991; Vendeville et al., 1987), shear zones (Jessell and Lister, 1991; Schrank et al., 2008), extensional fault systems (McClay and Scott, 1991), gravitational instabilities of mantle lithosphere (Pysklywec and Cruden, 2004), deformation-induced melt segregation (Barraud et al., 2001), and subduction (Faccenna et al., 1999; Funiciello et al., 2003; Jacoby, 1976; Kincaid and Olson, 1987; Schellart, 2008; Shemenda, 1994). Detailed reviews of the current state of the art in experimental tectonics include Buiter and Schreurs (2006) and Graveleau et al. (2012), and insightful discussions of the history of the field are given by Koyi (1997) and Ranalli (2001).

An analog model is only useful if its behavior is 'dynamically similar' to that of the natural prototype, so that experimental results can be extrapolated to geological length and time scales that are very different from those in the laboratory. However, the question of precisely what features of a model are required to ensure dynamical similarity is a subtle one that has exercised scientists for centuries. It has been recognized since antiquity that a model and its prototype must be geometrically similar, the classic example being the 'armillary sphere' model of the celestial sphere. In mechanics, however, geometrical similarity, while necessary, is not a sufficient condition for dynamical similarity. This seems first to have been recognized in the 1630s by Galileo, who noted that a large ship in drydock often collapses under its own weight whereas a smaller ship with exactly the same shape remains intact (Galileo, 1974). Galileo's insight has been further developed and formalized by many distinguished scientists (Barenblatt, 1996; Birkhoff, 1960; Bridgman, 1931; Buckingham, 1914; Vaschy, 1892). The essential result of this work can be simply stated as follows: two different instantiations of a mechanical system (e.g., a natural prototype and a scale model of it) are dynamically similar if they have the same values of all the dimensionless numbers ('groups') that are required to characterize their geometry, kinematics, and dynamics. Groups in the geometrical category include such things as aspect ratios (length/width), layer-depth ratios, and angles. Kinematical groups involve time in addition: examples are the ratios of two different time or velocity scales that are intrinsic to the system. Finally, dynamical groups contain physical parameters with units of mass. Examples are ratios of material properties such as densities or viscosities,

* Corresponding author. Tel.: +33 169153732.
E-mail address: ribe@fast.u-psud.fr (N.M. Ribe).

0040-1951/$ – see front matter
http://dx.doi.org/10.1016/j.tecto.2013.06.005

and ratios of two different forces or two different rates of energy production in the system. A classic example for a flowing fluid is the Reynolds number $\mathrm{Re} = \rho UL/\eta$, where U is the characteristic velocity, L is a characteristic length, ρ is the density, and η is the (dynamic) viscosity. The Reynolds number can be interpreted as the characteristic ratio of inertia ($\sim\rho U^2/L$ per unit volume) to viscous forces ($\sim\eta U/L^2$), or equivalently as the characteristic ratio of the rate of change of kinetic energy ($\sim\rho U^3/L$) to the rate of viscous dissipation ($\sim\eta U^2/L^2$). Another example is the Rayleigh number $\mathrm{Ra} = L^3\rho g\alpha\Delta T/\eta\kappa$ that governs thermal convection in a layer thickness L, where g is the gravitational acceleration, α is the coefficient of thermal expansion, ΔT is the temperature difference across the layer, and κ is the thermal diffusivity. It can be interpreted as the characteristic ratio of the rate of change of gravitational potential energy ($\sim\rho g\alpha U\Delta T$) to the rate of viscous dissipation ($\sim\eta U^2/L^2$), where the characteristic velocity scale is $U \sim \kappa/L$.

In the geological context, a particularly influential discussion of dynamical similarity was given by Hubbert (1937). According to Hubbert, similarity requires that all lengths, times and masses in the model are constant multiples of those in the prototype. Hubbert calls these multiplicative factors the 'model ratios' of length, time, and mass. The crucial point here is that these ratios are properties of the model/prototype pair as a whole, and are independent of position. Moreover, the constancy of these model ratios implies the existence of additional constant model ratios for derived quantities such as density, velocity, and force.

One of the cornerstones of Hubbert's (1937) approach is the principle that a scale model must have a 'mass distribution similar to that of the original' (p. 1468). Because this is equivalent to the requirement that the ratio of the densities at corresponding points in the model and the prototype be independent of position, we shall henceforth call it the 'density proportionality rule'. A corollary for the particular case of a system of layers with densities $\rho_1,\rho_2,\ldots,\rho_N$ is that each of the ratios $\rho_1/\rho_2,\rho_2/\rho_3,\ldots,\rho_{N-1}/\rho_N$ must be the same in both the model and the prototype. Hubbert's density proportionality rule has been followed by many subsequent authors. For example, Ramberg (1981, p. 4) states that dynamical similarity of a model to its prototype requires 'the condition of constant model ratio of mass per corresponding volume throughout the two structures'. In the same vein, Weijermars and Schmeling (1986, p. 328) state that 'Dynamic similarity of a model and a prototype … is only possible if … density ratios across subregions are similar'.

Here we show that density proportionality is in fact not a general requirement for dynamical similarity in systems involving slow (inertia-free) viscous flow, the limit most relevant for understanding geological structures. While density proportionality never violates dynamical similarity, it is not necessarily implied by the true dynamical similarity condition obtained from an analysis of the governing equations. This means that an insistence on density proportionality is unnecessarily restrictive in many cases, and that the range of analog materials that can be used to construct properly scaled models is wider than commonly supposed. We now illustrate these points by means of three examples.

2. Gravity tectonics

Many complex geological structures result from mechanical instabilities of superposed horizontal rock layers with different densities and mechanical properties. If one of the layers is overlain by another with higher density, gravitational (Rayleigh–Taylor) instability of the interface between them will occur, giving rise to structures such as diapirs and salt domes (Fig. 1a). The conditions for dynamical similarity in such systems can be determined by studying the simple model shown in Fig. 2a. Two layers with densities ρ_i, viscosities η_i, and initial thicknesses h_i ($i = 1, 2$) overlie an infinite halfspace with density ρ_3 and viscosity η_3. The upper surface is in contact with air or water of

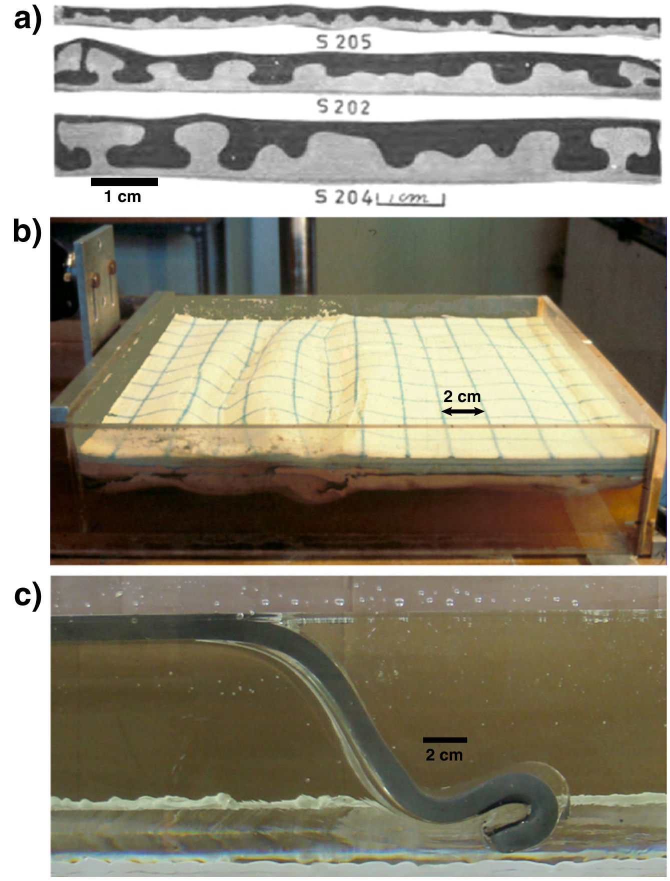

Fig. 1. Examples of experimental tectonics. (a) Diapirs due to gravitational (Rayleigh–Taylor) instability of two layers of silicone putty in a centrifuge (Ramberg, 1981). (b) Compressional deformation of a model continental lithosphere comprising sand, silicone putty and honey (photo P. Cobbold). (c) Free subduction of a sheet of silicone putty in a tank of glucose syrup (photo A. Davaille).

density ρ_0. This model configuration corresponds to model I-3 of Ramberg (1981, p. 58).

Define surface i as the lower surface of layer i, and let $\zeta_i(x,y,t)$ be the perturbed depth of that surface relative to its mean value, where (x,y) are horizontal Cartesian coordinates and t is time. Let z_i be the depth to surface i; thus $z_0 = \zeta_0$, $z_1 = h_1 + \zeta_1$, and $z_2 = h_1 + h_2 + \zeta_2$. Let $\mathbf{n}(x,y,t)$ be the unit vector normal to a surface, and let $\mathbf{t}(x,y,t)$ be any unit vector tangent to it. The surface indices on $\mathbf{n}$ and $\mathbf{t}$ are suppressed to simplify the notation.

Now define viscosity ratios $\gamma_{ij} = \eta_i/\eta_j$, the layer-depth ratio $h_{21} = h_2/h_1$, and the density contrasts $\Delta\rho_{ij} = \rho_i - \rho_j$. For definiteness, we suppose $\Delta\rho_{12} > 0$, so that surface 1 is gravitationally unstable. The density contrast $\Delta\rho_{01} < 0$, and $\Delta\rho_{23}$ can have either sign.

In the absence of inertia, the equations governing the conservation of mass and momentum in layer i are

$$\nabla\cdot\mathbf{u}^{(i)} = 0, \tag{1a}$$

$$\nabla P_i = \eta_i\nabla^2\mathbf{u}^{(i)}, \tag{1b}$$

where $\mathbf{u}^{(i)}$ is the velocity and P_i is the modified pressure, related to the total pressure p_i by $P_i = p_i - \rho_i gz$. The modified pressure P_i is not the same as the nonhydrostatic pressure, which is obtained by subtracting from p_i the pressure due to the weight of all the material above the point in question. The three components of $\mathbf{u}^{(i)}$ in the x-, y- and z-directions are $(u_1^{(i)},u_2^{(i)},u_3^{(i)})$.

We now write down the required boundary and matching conditions on the different surfaces. The upper surface $z = z_0$ is stress-free