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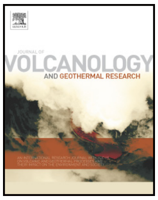

Are volcanic seismic *b*-values high, and if so when?

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ABSTRACT

The Gutenberg–Richter exponent b is a measure of the relative proportion of large and small earthquakes. It is commonly used to infer material properties such as heterogeneity, or mechanical properties such as the state of stress from earthquake populations. It is 'well known' that the b-value tends to be high or very high for volcanic earthquake populations relative to $b = 1$ for those of tectonic earthquakes, and that b varies significantly with time during periods of unrest. We first review the supporting evidence from 34 case studies, and identify weaknesses in this argument due predominantly to small sample size, the narrow bandwidth of magnitude scales available, variability in the methods used to assess the minimum or cutoff magnitude M_c, and to infer b. Informed by this, we use synthetic realisations to quantify the effect of choice of the cutoff magnitude on maximum likelihood estimates of b, and suggest a new work flow for this choice. We present the first quantitative estimate of the error in b introduced by uncertainties in estimating M_c, as a function of the number of events and the b-value itself. This error can significantly exceed the commonly-quoted statistical error in the estimated b-value, especially for the case that the underlying b-value is high. We apply the new methods to data sets from recent periods of unrest in El Hierro and Mount Etna. For El Hierro we confirm significantly high b-values of 1.5–2.5 prior to the 10 October 2011 eruption. For Mount Etna the b-values are indistinguishable from $b = 1$ within error, except during the flank eruptions at Mount Etna in 2001–2003, when $1.5 < b < 2.0$. For the time period analysed, they are rarely lower than $b = 1$. Our results confirm that these volcano–tectonic earthquake populations can have systematically high b-values, especially when associated with eruptions. At other times they can be indistinguishable from those of tectonic earthquakes within the total error. The results have significant implications for operational forecasting informed by b-value variability, in particular in assessing the significance of b-value variations identified by sample sizes with fewer than 200 events above the completeness threshold.

1. Introduction

Volcanic earthquakes provide insight into physical processes acting at volcanoes, such as the mechanisms of deformation of the volcanic edifice and magma accumulation, and statistical analysis of earthquake catalogues are a key component of eruption forecasting methods (McNutt, 1996). Increased rates of earthquakes are a primary indicator of volcanic unrest, and changing locations of earthquake hypocentres can be used to map magma migration (Wiemer and Wyss, 2002). The frequency–magnitude distribution (FMD) of volcanic earthquakes can provide insight into the state of stress or material properties, and are a key component of most studies of volcanic seismicity.

Where the catalogue is completely reported, the FMD, commonly takes the form of a Gutenberg–Richter (GR) relation (Gutenberg and Richter, 1954):

$$\log(N) = a - bM, \qquad (1)$$

where N is the total number of earthquakes of magnitude equal to or greater than M, and a and b are real, positive constants characteristic of the specific catalogue. The parameter a is the logarithm of the number of earthquakes with $M \geq 0$, and is thus a measure of the seismicity rate of the region. The b-value represents the relative proportion of large and small events in the catalogue. It is best calculated or inferred using the maximum likelihood method (Aki, 1965), now used almost universally in earthquake seismology (Mignan and Woessner, 2012). Other methods such as a least squares fit of the data to Eq. (1) are known to produce a biassed estimate (Naylor et al., 2010). In addition, if the bandwidth of data is narrow, or equivalently the sample is small, then it is easy to overestimate the underlying b-value (Main, 2000). Finally, the b-value may also be biassed due to incorrect identification of the threshold for complete reporting, denoted M_c here (Mignan and Woessner, 2012). These and other sources of bias introduce an epistemic error to any inference from the data. In principle this should be accounted for in addition to the aleatory uncertainties inferred from the random error associated with measurement or statistical fluctuation in the data, but it is often neglected in studies of volcanic earthquake populations.

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The Gutenberg–Richter form of the distribution holds, at least for small and intermediate events across a remarkable range of sizes and loading conditions, from laboratory experiments to volcanic and tectonic earthquakes (Main, 1996). In controlled laboratory tests, seismic *b*-values commonly change systematically with respect to a variety of controlling factors. These include the degree of material heterogeneity (Mogi, 1962), the level of applied stress (Scholz, 1968), the degree of stress concentration, i.e. the stress intensity normalised to the fracture toughness (Meredith and Atkinson, 1983), the chemical reactivity of the pore fluid (Meredith and Atkinson, 1983), and the pore fluid pressure (Sammonds et al., 1992). In nature other factors that affect the *b*-value systematically include the earthquake focal mechanism (Schorlemmer et al., 2005), the depth (Mori and Abercrombie, 1997), and the degree of coupling or strain partition between seismic and aseismic deformation at plate boundaries (Mazzotti et al., 2011).

The *b*-value for tectonic earthquakes, using best practice and large regional or global data sets, is commonly reported as taking values near unity (Frolich and Davis, 1993). In contrast the reported *b*-values from published studies of earthquake populations associated with volcanic unrest are commonly reported as being significantly higher than this, allowing for the random error expected for a *b*-value of unity (described in more detail below). The main question we address here is whether this difference is real or, at least to some extent, an artefact of the known sources of bias described above.

To examine this question we first use synthetic data to explore the effect of various factors on the estimated *b*-value, denoted $\tilde{b}$, and the underlying *b*-value, henceforth denoted *b*. Uncertainties in $\tilde{b}$ at one standard deviation, denoted $\sigma_{\tilde{b}}$, are estimated using the method of Shi and Bolt (1982), which correctly reflects the (approximately) Poisson 'counting errors' expected from sampling a whole number of events (Greenhough and Main, 2008). The advantage of using synthetic data is that we can distinguish between the random error $\sigma_{\tilde{b}}$, and the systematic error or bias $\tilde{b}-b$, or equivalently to errors of precision and accuracy respectively. We show how both depend intrinsically on the sample size. First we determine an optimum method of estimating the cutoff magnitude of complete reporting of events, M_c, for catalogues of different sizes, and then propose a formal workflow for the estimation of M_c. The proposed workflow is then applied to two volcanic seismic catalogues at Mount Etna and El Hierro as important examples of recently-active volcanic systems to address the questions: (a) are the *b*-values higher than 1? And (b) do they vary with time significantly outside the estimated margins of error? For these examples, *b* is remarkably stationary and similar to (~1) or only somewhat larger (1–1.5) than to those of tectonic earthquakes, except for specific transients where the *b*-value can be significantly greater than background at 95% confidence. The results presented here will provide greater confidence in identifying statistically-significant variations in *b*-value, and in identifying physical causes for this variability.

2. Review and synthesis of previous studies

In this section we extend the review of McNutt (2005), who summarised reported *b*-values and associated parameters such as source depth from 13 different volcanoes around the world. This review includes *b*-values as high as 3 in one case (McNutt, 2005). In Table 1 we extend this study to 21 volcanoes, and include a wider range of

Table 1
Compilation of *b*-values and range of magnitudes for volcanic seismic catalogues.

Reference	Volcano	Dates	Depth, km	*N*	Method M_c	Mag. range	Method *b*	b_{min}	b_{typ}	b_{max}
Jacobs and McNutt (2010)	Augustine	2000–2006	−2−0	100	ZMAP	–	MLE	0.8	1.4	2.1
Jacobs and McNutt (2010)	Augustine	17/11/05–10/12/05	−2–0	~250	ZMAP	−0.1–0.7	MLE	–	–	1.85
M. Wyss (written comm.)	Coso		0.8–3					–	–	1.7
Ibanez et al. (2012)	El Hierro	19/7/11–16/9/11	8–16	7000+	90GFT	1.3–2.7	LS	1.12	1.57	2.25
Ibanez et al. (2012)	El Hierro	19/07/2011	8–16	200	90GFT	–	LS	0.75	1.25	2.55
Marti et al. (2013)	El Hierro	14/8/11–18/8/11	8–16	–	–	–	MLE	0.8	1.1	2.3
Ibanez et al. (2012)	El Hierro	19/7/11–28/7/11	8–16	–	90GFT	1.5–2.6	LS	0.81	1.2	3.01
Patane et al. (1992)	Etna	1984	–	200	–	2.8–	MLE	0.8	1.1	1.7
Patane et al. (1992)	Etna	29/3/1983–6/8/1983	–	–	–	2.5–	MLE	0.7	1.0	2.1
Murru et al. (1999)	Etna	–	9–15	50	MaxC	2.5–	MLE	1.4	1.5	3.5
Centamore et al. (1999)	Etna	1/1/1990–31/12/92	–	100	–	2.3–5.1	LS	0.5	1.2	1.9
Centamore et al. (1999)	Etna	1/1/1990–31/12/92	–	100	–	2.3–5.1	MLE	0.9	1.1	1.7
Murru et al. (2007)	Etna	July–Aug 2001	0–2	50	GFT	2.6–3.5	MLE	0.7	1	2.6
Murru et al. (2005)	Etna	July–Aug 2001	0–12	50	90GFT	2.6	MLE	0.8	1.5	2.50
Murru et al. (2007)	Etna	Aug 1999–Dec 2005	1–3	100	90GFT	2.5	MLE	0.7	1.0	1.86
Sanchez et al. (2005)	Galeras	Sep 1995–Jun 2002	0–2	300	-	1.2–2.8	MLE	0.65	1.0	1.4
Jolly and McNutt (1999)	Katmai	–	6–8	–	–	–	–	1.0	1.3	1.6
Wyss et al. (2001)	Kilauea	-	4–7,20	–	–	–	–	–	–	1.9
Wyss et al. (2001)	Kilauea	1979–1997	4–7	50	-	1.8–2.6	MLE & LS	0.6	1.0	1.73
Wiemer et al. (1998)	Long Valley	1989–1998	1–11	150	MaxC	1.3–	MLE	1.1	1.4	2.0
Jolly and McNutt (1999)	Mageik	Sep 1996–April 1997	0–5	–	–	–	WLS	1.0	1.5	2.0
Bridges and Gao (2006)	Makushin	July 1996–April 05	0–8	50	74GFT	0.9–3.9	MLE	0.73	1.21	2.03
Wiemer et al. (1998)	Mammoth Mtn.	1989–1990.5	3–4,7–9	150	MaxC	1.3–	MLE	0.95	1.2	1.6
Jolly and McNutt (1999)	Martin/Mageik	Sep 1996–April 1997	−2–10	–	–	0.7–4.5	WLS	–	–	1.56
Wiemer and McNutt (1997)	Mount Spurr	1991–1995	2.3–4.5	100	Inspection	0.1–2.2	MLE & LS	0.6	1.1	1.8
Main (1987)	Mount St Helens	20/3/80-18/5/80	na	~300	Inspection	3.5–5	MLE	0.5	1.0	1.5
Wiemer and McNutt (1997)	Mount St. Helens	1988 - Jan 1996	2.7–3.8	100	Inspection	0.4–2.8	MLE & LS	0.8	1.2	1.6
Wyss et al. (1997)	Off-Ito	1982–1996	7–15	100	MaxC	1.6–2.5	MLE	0.44	1.0	1.54
M. Wyss (written comm.)	Oshima		4					–	–	1.5
Sanchez et al. (2004)	Pinatubo	29/6/99-19/7/99	0–4,8–13	100	ZMAP	0.73–	MLE	1.0	1.3	1.7
Novelo-Casanova et al. (2006)	Popocatepetl	Dec 2000–Jan 2001	2–7	20	Inspection	1.9–3.3	MLE	1.0	1.7	2.70
S. Wiemer (written. comm.)	Redoubt		3–4,6–8					–	–	1.7
Power et al. (1998)	Soufriere Hills	Aug 1995–Mar 1996	2.0–2.5	100	–	1.7–2.4	MLE	0.9	1	3.07
Farrell et al. (2009)	Yellowstone	1984–2006	4–18	>10	EMR	1.5–	MLE	0.5	1.0	1.5

Values for *N* are the number of events analysed in each catalogue. These figures are either given or estimated from figures. The methods for calculating the completeness magnitude, M_c, are; using ZMAP software; the goodness-of-fit method (GFT) with given percentage threshold (e.g. 90GFT is 90% fit); the Maximum Curvature method (MaxC); Inspection is choosing a M_c by eye; and using the Entire Magnitude Range method (EMR). The methods for approximating the *b*-value are the Maximum Likelihood Estimation (MLE) and the least squares and weighted least squares fit (LS & WLS). The *b*-value ranges in each study are described by the minimum (b_{min}) and maximum (b_{max}) quoted values in the study, with a typical value (b_{typ}) being estimated by eye.