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Fall velocity of multi-shaped clasts

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ABSTRACT

Accurate settling velocity predictions of differently shaped micro- or macroclasts are required in many branches of science and engineering. Here, a single, dimensionally correct equation is presented that yields a significant improvement on previous settling formulas for a wide range of clast shapes. For smooth or irregular clasts with known axial dimensions, a partially polynomial equation based on the logarithmic values of dimensionless sizes and settling velocities is presented, in which the values of only one coefficient and one exponent need to be adapted for different shapes, irrespective of the Reynolds number. For irregular, natural clasts with unknown axial dimensions, a polynomial equation of the same form is applied, but with different coefficients. Comparison of the predicted and measured settling velocities of 8 different shape classes as well as natural grains with unknown axial dimensions in liquids, representing a total of 390 experimental data points, shows a mean percentage error of −0.83% and a combined R^2 value of 0.998. The settling data of 169 differently shaped particles of pumice, glass and feldspar falling in air were also analyzed, which demonstrates that the proposed equation is also valid for these conditions. Two additional shape classes were identified in the latter data set, although the resultant equations are less accurate than for liquids. An Excel spreadsheet is provided to facilitate the calculation of fall velocities for grains settling individually and in groups, or alternatively to determine the equivalent sieve size from the settling velocity, which can be used to calibrate settling tubes.

1. Introduction

The settling velocity of micro- and macroclasts has wide applications in science and engineering, including mineral processing (Concha and Almendra, 1979; Ofori-Sarpong and Amankwah, 2011), tailings segregation (Talmon et al., 2014), materials engineering (Purmajidian and Akhlaghi, 2014), powder technology (Bullard and Garboczi, 2013), atmospheric science (Chen and Fryrear, 2001; Siewert et al., 2014), and food engineering (Koyama and Kitamura, 2014). In the life sciences it has been applied, inter alia, to evolutionary microbiology (Briguglio and Hohenegger, 2011) and phytoplankton distribution (Greenwood and Craig, 2014), whereas in earth sciences it has been used to predict sediment transport (Le Roux, 1997a, 1998, 2001, 2005), as well as to model the fluid dynamics of magma chambers (Gutiérrez and Parada, 2010) and the aerodynamic behavior of volcanic particles, e.g. in dilute pyroclastic density currents (Choux et al., 2004; Dellino et al., 2004; Alfano et al., 2011; Douillet et al., 2014).

In the geothermal energy field, the settling velocity of particles also has practical uses (Bonham, 1994). Some high temperature geothermal brine resources, e.g. in the Imperial Valley of California, contain large amounts of geothermal energy, but may not be suitable for the commercial production of electricity because of a high dissolved silica content that precipitates on the walls of pipes within the power production plants. In the past, some power plants used flash crystallizer/reactor clarifier technology to precipitate the silica into sludge that is removed from the reactor clarifiers and pressed into a filter cake for disposal. However, there is still some precipitation growing on the walls of vessels and pipes, which tends to break loose in particulate scales to join the silica particles in the geothermal brine flowing through the plant. This material, which intermingles with the particulate silica in the sludge and is eventually incorporated into the sludge filter cake, contains hazardous substances such as As, Pb, Cu, Zi and Ag, thus restricting its disposal as simple landfill. A solution to this problem is to separate the toxic scale particulates from the non-toxic particulate silica by vertical separation, effected by means of a continuous decantation process in which a liquid slurry of silica and scale particulates is pumped upward at a velocity greater than the terminal settling velocity of the silica and less than the terminal settling velocity of the toxic scale. This process minimizes toxic waste disposal problems and enables a large amount of particulate silica waste to be disposed of as ordinary landfill (Bonham, 1994). However, in order to fine-tune the separation process, it is necessary to accurately predict the settling velocities of the silica and toxic particulates.

Here, a single equation is proposed that gives accurate results from the Stokes range (quartz spheres less than 0.005 cm in diameter settling in water at 20 °C) to quartz spheres up to at least 22 cm settling under the same conditions. In addition, the same equation can be used to accurately predict the group settling velocity of natural clay- to gravel-sized sediments by simply adapting the coefficients.

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2. Previous work

Numerous papers have been written on the settling velocity of differently shaped clasts (e.g. Stokes, 1851; Wadell, 1932; Rubey, 1933; Rouse, 1936; Corey, 1949; Janke, 1965, 1966; Gibbs et al., 1971; Warg, 1973; Komar and Reimers, 1978; Komar, 1980; Baba and Komar, 1981a,b; Dietrich, 1982; Le Roux, 1992, 1996; Cheng, 1997; Le Roux, 1997a,b, 2002a,b, 2004; Ferguson and Church, 2004). The most widely used single equations have probably been those of Dietrich (1982), Cheng (1997) and Ferguson and Church (2004), although none of these are very accurate over a wide range of clast shapes and sizes. Le Roux (1992) proposed a set of 5 equations for spheres of any size and density settling in fluids of different densities and viscosities, which are accurate to within 2% for the very well constrained data set of Gibbs et al. (1971). The obtained sphere settling velocity was subsequently incorporated into an equation for clasts with shapes varying from spheroids to ellipsoids, discs and cylinders (Le Roux, 2004). Nevertheless, the use of 5 different equations to calculate sphere settling velocities remains awkward.

In this paper, the settling equations of Dietrich (1982), Cheng (1997) and Ferguson and Church (2004) are compared with the results of the present study.

Dietrich (1982) defined a dimensionless sphere size as

$$D_{dn*} = \frac{\rho_\gamma g {D_n}^3}{\rho \nu^2}, \tag{1}$$

where ρ_γ is the submerged grain density (the grain density ρ_s minus the fluid density ρ), g is the acceleration due to gravity, D_n is the nominal grain diameter, and ν is the kinematic viscosity $\left(\frac{\mu}{\rho}\right)$, μ being the dynamic viscosity. This author also defined a dimensionless settling velocity as

$$U_{dwn*} = \frac{\rho {U_{wn}}^3}{\rho_\gamma g \nu}. \tag{2}$$

where U_{wn} is the actual settling velocity. His settling equation for spheres was given as

$$\log_{10} U_{dwn*} = -3.76715 + 1.92944(\log_{10} D_{dn*}) - 0.09815(\log_{10} D_{dn*})^2 - 0.00575(\log_{10} D_{dn*})^3 + 0.00056(\log_{10} D_{dn*})^4. \tag{3}$$

According to Dietrich (1982), Eq. (3) should not be used for values of D_{dn*} smaller than 0.05 or greater than 5×10^9.

For clasts of any size, shape, density and roundness, Dietrich proposed the following equation:

$$U_{dwp} = \left[0.65 - \frac{CSF}{2.83}\tanh(\log D_{dn*} - 4.6)\right]^{\left(\frac{1+3.5-P}{2.5}\right)} 10^{\left[\log_{10} U_{dwn*} + \left(\log\left(1-\frac{1-CSF}{0.85}\right)\right) - (1-CSF)^{2.3}\tanh(\log_{10} D_{dn*} - 4.6) + 0.3(0.5-CSF)(1-CSF)^2(\log_{10} D_{dn*} - 4.6)\right]}, \tag{4}$$

where P is the Powers roundness scale and CSF is the Corey (1949) shape factor given by

$$CSF = \frac{D_s}{\sqrt{D_i D_l}}, \tag{5}$$

D_s, D_i and D_l being the short, intermediate and long orthogonal grain axial diameters, respectively.

For smooth clasts, $P = 6$, whereas for natural grains, Dietrich proposed that $P = 3.5$ and the $CSF = 0.7$.

Cheng (1997) derived an equation for the settling velocity of natural grains based on the data of Zhu and Cheng (1993):

$$U_{wp} = \frac{\nu\left(\sqrt{25 + 1.2{D_*}^2} - 5\right)^{3/2}}{D_n}, \tag{6}$$

where

$$D_* = D_n\left(\frac{\rho_\gamma g}{\nu^2}\right)^{1/3}. \tag{7}$$

Ferguson and Church (2004) defined the following equation to determine the settling velocity of particles:

$$U_{wp} = \frac{\rho_\gamma g {D_n}^2}{C_1 \nu + \sqrt{0.75 C_2 \rho_\gamma g {D_n}^3}}. \tag{8}$$

For smooth spheres, $C_1 = 18$ and $C_2 = 0.4$, for natural grains $C_1 = 18$ and $C_2 = 1$ if sieve diameters are used, and for other shapes or where nominal diameters are used for natural grains, $C_1 = 20$ and $C_2 = 1.1$.

3. Methodology

Water and air are both Newtonian fluids, i.e. they deform continuously and permanently when subjected to external forces. It is therefore convenient to use a non-dimensional approach, which means that dimensionally correct equations that are accurate for Newtonian liquids will also be accurate for Newtonian gases, provided that all the significant variables are taken into account (Middleton and Southard, 1984). The variables playing a role in particle settling velocity include the particle diameter and density, the fluid density and dynamic viscosity, as well as the particle shape (ψ), where various expressions or combinations of the three orthogonal axes can be used. The particle and fluid density can be combined into a submerged particle density, which leaves only the variables U_w, g, D_n, ρ_γ, μ, and ψ. Considering only spheres for the moment, ψ can be left out, so that the remaining variables can be combined into a dimensionless nominal grain diameter given by

$$D_{dn} = D_n \sqrt[3]{\frac{\rho g \rho_\gamma}{\mu^2}}. \tag{9}$$

In the same way, a dimensionless settling velocity can be defined for spheres by

$$U_{dwn} = U_{wn} \sqrt[3]{\frac{\rho^2}{\mu g \rho_\gamma}}. \tag{10}$$

The settling of particles is controlled by shear stress and pressure exerted by the fluid on its surface, called the viscous drag and pressure or form drag, respectively (Middleton and Southard, 1984). In the case of small spheres settling at very low Reynolds numbers, given by

$$\mathrm{Re}_w = \frac{\rho D U_{wn}}{\mu}, \tag{11}$$

Stokes' law of settling (Stokes, 1851) can be used to precisely calculate the fall velocity:

$$U_{wn} = \frac{{D_n}^2 g \rho_\gamma}{18\mu}. \tag{12}$$

The upper limit of Stokes' law of settling is not clear: there is a gradual deviation of the observed settling velocity from the latter, beginning at a Reynolds number which most have considered to be 1, although