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A multiplier-based method of generating stochastic areal rainfall from point rainfalls

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ABSTRACT

Catchment modelling for water resources assessment is still mainly based on rain gauge measurements as these are more easily available and cover longer periods than radar and satellite-based measurements. Rain gauges however measure the rain falling on an extremely small proportion of the catchment and the areal rainfall obtained from these point measurements are consequently substantially uncertain. These uncertainties in areal rainfall estimation are generally ignored and the need to assess their impact on catchment modelling and water resources assessment is therefore imperative. A method that stochastically generates daily areal rainfall from point rainfall using multiplicative perturbations as a means of dealing with these uncertainties is developed and tested on the Berg catchment in the Western Cape of South Africa. The differences in areal rainfall obtained by alternately omitting some of the rain gauges are used to obtain a population of plausible multiplicative perturbations. Upper bounds on the applicable perturbations are set to prevent the generation of unrealistically large rainfall and to obtain unbiased stochastic rainfall. The perturbations within the set bounds are then fitted into probability density functions to stochastically generate the perturbations to impose on areal rainfall. By using 100 randomly-initialized calibrations of the AWBM catchment model and Sequent Peak Analysis, the effects of incorporating areal rainfall uncertainties on storage-yield-reliability analysis are assessed. Incorporating rainfall uncertainty is found to reduce the required storage by up to 20%. Rainfall uncertainty also increases flow-duration variability considerably and reduces the median flow-duration values by an average of about 20%.

1. Introduction

Rainfall is a major driver of hydrological processes and therefore a major component of models of these processes. Although rainfall measurement by remote sensing provides better spatial coverage than rain gauge measurements, these data are generally not as readily available as rain gauge measurements and typically span over shorter periods. Rain gauge measurements are therefore still a major source of rainfall data for practical hydrological analysis including areal rainfall determination. They are also vital for verifying remotely-sensed rainfall measurements. Rain gauges are usually sparsely located and a rain gauge measures rainfall over a very small area of the catchment. The resulting areal rainfall estimates are therefore substantially uncertain irrespective of the areal rainfall estimation method used. Although areal rainfall uncertainty has been the focus of many studies (Seed and Austin, 1990; Andreassian et al., 2001; Vrugt et al., 2008; Volkmann et al., 2010), there is still a lack of methods that are sufficiently robust yet simple enough for routine practical application. The aim here was to develop such a method and assess it using storage-yield-reliability and flow-duration analysis – both typical water resources assessment problems. It was also the intention to keep the formulation simple and robust so as to make it easily applicable in practice.

The main basis of the approach is the understanding that the uncertainty in areal rainfall determination can be obtained from the differences in areal rainfall obtained if half (or about half) of the rainfall stations are alternately omitted whilst trying to maintain as uniform a catchment-wide spatial coverage as possible. Since the best estimate of areal rainfall is that obtained when all the rain gauges are used, the uncertainties obtained at half the rain gauge density would need to be scaled down before being applied at the actual density. An approach for scaling down the perturbations is therefore included in the formulation. The approach proposed has the advantage of naturally adapting to rain gauge data availability and any unique spatial rainfall patterns of the area because it uses the actual data that has been recorded which implicitly incorporates these patterns or characteristics. The approach is non-parametric therefore has the advantage of "letting the data speak for themselves" (Wand and Jones, 1995) which parametric method typically lack. The proposed method also allows for any method of estimating areal rainfall as the uncertainties are based on areal rainfall estimates. Furthermore, the

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approach enables the inaccuracies of the areal rainfall estimation method to get naturally incorporated in the uncertainty framework. The assessment of areal rainfall uncertainty by varying rain gauge density has been studied previously (Cheng et al., 2012, 2007; Anctil et al., 2006; Andreassian et al., 2001; Duncan et al., 1993) but these studies did not specifically aim at formulating simple methods for incorporating uncertainties on areal rainfall estimates. Hydrologic uncertainty has been expressed multiplicatively in several other studies (e.g. Kavetski et al., 2006; Vrugt et al., 2008; Kuczera et al., 2010; McMillan et al., 2011) and this approach is adopted here. The differences in areal rainfall are expressed as ratios (here after called multipliers).

Table 1
Basic statistics of 8 rain gauge stations in the study catchment.

Rain gauge station	MAR (mm/year)	% of rain days per year
0041713 W	643	9
0041417 W	411	16
0041060 W	578	20
0021548 W	564	31
0021130 A	367	15
0021105 W	456	22
0021230 W	615	21
0021621 W	620	26

2. Case study catchment

Since the modelling approach is data-based, it was decided to search for a reliable daily rainfall record that required very little patching. The catchment selected also needed to have a reliable natural streamflow record to enable rainfall–runoff modelling for assessing the effect of incorporating rainfall uncertainties on storage-yield-reliability and flow-duration relationships. Considering these factors, the 1280 km^2 Berg catchment up to river gauging station G2H014 in South Africa (Fig. 1) was selected for the analysis. Daily rainfall data were obtained from Lynch (2003) and required an average of 3% of patching while streamflow and evaporation data were obtained from the South African Department of Water Affairs webpage (www.dwa.gov.za). Four of the rain gauge stations (0041713 W, 0021130 W, 0041417 W and 0021230 W) had daily rainfall data available over a 42 year long period (1939–1981) while the remaining four had data for the 4 year period between 1967 and 1971. Table 1 shows the basic statistics of the 8 rainfall stations for the 4 year period (1967–1971) for which all 8 had rainfall daily data available. Daily streamflow data at G2H014 (Fig. 1) were available between 1967 and 1981 although there were considerable periods with missing flows. A 4-year record of daily evaporation from 1980 to 1984 was available from station H1E002.

3. Development of stochastic areal rainfall generator

3.1. Multipliers and their characteristics

If $L_{1,t}$ and $L_{2,t}$ are non-zero areal rainfall estimates obtained using half the rain gauges by alternately omitting half of them whilst trying to maintain as uniform a catchment-wide spatial coverage as possible, two multipliers of areal rainfall $M_{1,t}$ and $M_{2,t}$ that are reciprocals of each other can be obtained for period t as:

$$M_{1,4} = L_{1,4}/L_{2,t} \quad \text{and} \quad M_{2,t} = L_{2,t}/L_{1,t} \tag{1}$$

The relevant characteristics of these multipliers need to be identified for them to be applied effectively for incorporating uncertainty. An expected feature of the multipliers is their variation with rainfall magnitude because higher rainfall cover larger areas and the relative differences recorded by the rain gauges would be lower in comparison to lower rainfall. There is also the expectation that the multipliers would be less varied and closer to unity as the rain gauge density increases as the computed values of $L_{1,t}$ and $L_{2,t}$ would be closer if they are determined from a more closely spaced rainfall stations. These two aspects; the variation of the multipliers with rainfall magnitude and with rain gauge density were investigated using the rainfall data for the 8 stations for the period April 15 1967 to January 5 1971. To do this, multipliers were computed for the following 3 cases:

> *Case A*: Areal rainfall from all 8 stations related to multipliers obtained using 0041713 W, 0041060 W, 0021548 W and 0021230 W as one group and 0041417 W, 0021130 A, 0021105 W and 0021621 W as the other.
> *Case B*: Areal rainfall from 4 stations; 0041417 W, 0021105 W, 0021130 A and 0021621 W related to multipliers obtained using 0041417 W and 0021105 W as one group and 0021130 A and 0021621 W as the other.
> *Case C*: Areal rainfall from 4 stations; 0041713 W, 0021230 W, 0041060 W and 0021548 W related to multipliers obtained using 0041713 W and 0021230 W as one group and 0041060 W and 0021548 W as the other.

Fig. 2 shows the variation of multiplier values with areal rainfall. A few multipliers exceeded a value of 5 but these were ignored and considered too large to be applied. Fig. 2 shows the expected

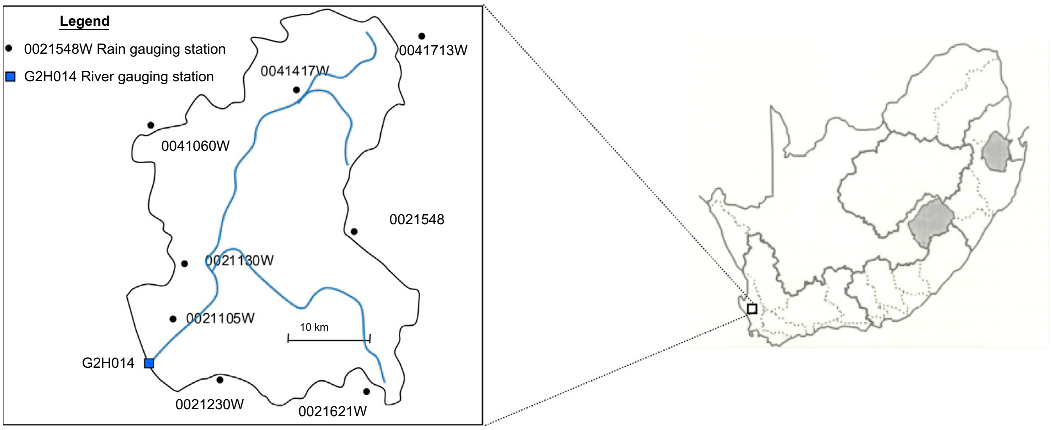

Fig. 1. The Berg River catchment to G2H014.

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