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Research Paper

# Expression for ETC of the solid phase of randomly packed granular materials

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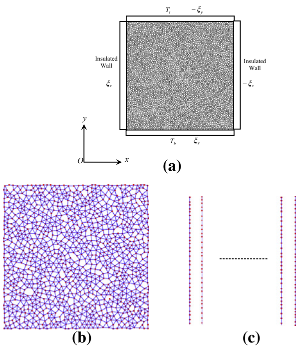
## HIGHLIGHTS

- An expression for ETC of the solid phase of granular material is derived.
- ETC is expressed in terms of grain size distribution and compressive strain.
- The predicted ETC is compared with the numerical result obtained by FEM.

## GRAPHICAL ABSTRACT

(a) The schematic of thermal and mechanical boundary conditions imposed on the granular sample. (b) The schematic of pipe-network model. (c) Schematic of the parallel-column model.



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## ABSTRACT

Effective thermal conductivity (ETC) is an important parameter describing the thermal behavior of particulate materials, and has been extensively examined in the past decades. In this paper, a theoretical model called parallel-column model is proposed and based on this model, an analytical expression is developed to predict the ETC of the solid phase of granular materials. The ETC is expressed in terms of particle size distribution, the compressive strain and thermal conductivity of individual particle. Different types of contact force models and grain size distribution is incorporated into the parallel-column model and the expression for ETC is derived for different types of contact force models and grain size distribution.

## 1. Introduction

The granular material consists of randomly packed, different sized particles in contact with each other and immersed in a connected uniform matrix which may be fluid or gas. The thermal conduction phenomena in granular material has been encountered in many engineering and scientific applications, such as geotechnical engineering application [1] and powder metallurgy [2].

Some heat transfer processes may exist in the granular material, e.g. the solid phase conduction between particles and particle-interstitial gas thermal conductions, convection, and radiation [3]. Referring to the work on assessment of heat transfer mechanisms in granular materials by Argento and Bouvard [4], if the thermal conductivity of the particle material (for example steel, 46.52 W/(m K)) is $2.0 * 10^3$ times as large as that of the interstitial gas phase, the heat transfer from gas and exposed surface accounts

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**Nomenclature**

| | |
|---|---|
| $T$ | temperature, °C |
| $r$ | particle radius, m |
| $d$ | particle diameter, m |
| $\delta^n$ | normal relative contact displacement, m |
| $N$ | number of particles in the granular sample |
| $N_1$ | number of particles in one pipe column |
| $\alpha$ | contact angle |
| $k$ | thermal conductivity of the particle, W/m K |
| $a$ | contact radius, m |
| $k^n$ | normal contact stiffness coefficient, N/m |
| $f^n$ | normal contact force, N |
| $L$ | height of the granular assembly, m |
| $D$ | width of the granular assembly, m |
| $F$ | force applied on one particle column, N |
| $\xi$ | compressive displacement applied on the granular assembly, m |
| $E$ | characteristic modulus, N/m$^2$ |
| $n_p$ | number of pipe columns in the granular sample |
| $\lambda_e$ | effective thermal conductivity of the granular material, W/m K |
| $\varepsilon_{xx}$ | compressive strain along the $x$ axis |
| $\varepsilon_{yy}$ | compressive strain along the $y$ axis |
| *Subscripts* | |
| $i, j$ | particle $i$, particle $j$ |
| $l$ | left side |
| $r$ | right side |
| $m$ | minimum |
| $M$ | maximum |

for only a small portion and these heat transfer mechanisms are largely dominated by solid phase conduction. Based on above analysis, the gas phase is not considered in this paper. Only solid phase conduction between particles is taken into account.

Chan and Tien [5] presented an analytical study for the heat transfer through the solid phase of a packed bed of spheres with regular packing structure. A regularly packing structure is one in which the same arrangement of particles, uniform in size, is repeated throughout the granular material. However, as highlighted in the literature [6], regular arrangements do not reliably reflect the behavior of the granular material with an irregular packing microstructure.

For randomly packed granular material, the structure of the granular material is very complex, consisting of different grain sizes and geometries. A detailed prediction of the effective thermal conductivity of this heterogeneous media requires a knowledge of the shape, size, location (distribution) and conductivity of each particle in the system together with interaction between particles [7]. Such information is difficult to represent, which often challenge the accurate estimation and interpretation of the effective thermal conductivity of granular material.

A few works [8,9] used the well-established finite element method (FEM) to discretize each particle and conducted thermal analysis for granular material with small particle counts. However, modeling heat transfer in granular material that contain large numbers of particles remains a challenge due to an excessively large scale finite element model resulting from the discretization of each particle.

On the other hand, some attempts termed as thermal discrete element method (TDEM) [10–12] have been made in which the particles are individually represented as an isothermal disk/sphere. Although simple and computationally efficient, the isothermal assumption made in the model over simplifies the actual thermal behavior of particles.

One of the disadvantages of finite element method (FEM) and thermal discrete element method (TDEM) is that the shape, size, location (distribution) and conductivity of each particle is required [7]. Such information is difficult to represent because for randomly packed granular material, the structure of the granular material is very complex, consisting of different grain sizes and geometries.

Effective thermal conductivity (ETC) is an important parameter describing the thermal behavior of particle systems. A number of correlations for ETC exist in the literature. Yun and Santamarina [13] divided previous studies into two categories: semi-empirical correlations and theoretical mixture correlations. Semi-empirical correlations [14,15] are extracted from thermal conductivity experimental data sets. Many parameters are included in the expression for the thermal conductivity. These parameters are determined by fitting with the experimental data. Theoretical mixture correlations [16] are derived from basic physical principles and contain no empirical parameters. The effective thermal conductivity is expressed in term of the volume fraction and the thermal conductivity of the components. As pointed out by Abyzov [17], theoretical mixture correlations do not include details of real systems (for example, particle size).

Liang and Li [18,19] propose a parallel-column model for the heat transfer through the solid phase of the randomly packed granular material and derive an expression for the effective thermal conductivity (ETC) within the framework of the parallel-column model. Based on Liang and Li [18,19], this paper performs further study on the expression for ETC.

Firstly, different types of contact force model at the contact point between particles are incorporated into the parallel-column model and the expressions for ETC are further deduced. Numerical simulations are performed to illustrate the accuracy of these expressions. Secondly, as pointed out in the literature [12,20], the isotropy of the medium is a crucial point to ensure that the effective properties are independent of the direction. In the progress of deriving the expression for ETC, the assumption of the isotropy of the random granular packing is used. Here, the contact angles distribution of the random granular packing is plotted and discussed to verify the assumption of the isotropy of the particle packing.

The paper is outlined as follows. Section 2 lists the various steps in the process of deriving the expression for ETC. Section 3 is dedicated to the numerical modelling of granular systems and some comparisons are performed between FEM and the expression for ETC to the investigation of the accuracy of the expression. Discussions are inferred in Section 4. Conclusions are inferred in Section 5.

## 2. Mathematical model

Consider a granular assembly subjected to the thermal ($T_l$ on the left side and $T_r$ on the right side) and mechanical (compressive load along both x axis and y axis) boundary condition, as shown in Fig. 1, in which each particle may be in contact with a number of adjacent particles, and heat is conducted through the contact zones between the particles. We want to compute the effective thermal conductivity of this granular assembly.