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Research Paper

Geometric optimization and performance study of a constructal T-shaped fin under simultaneous heat and mass transfer

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HIGHLIGHTS

- Analysis for constructal T-shaped fin under dehumidifying conditions.
- Thermal conductivity and heat transfer coefficient dependent on temperature.
- Two approximate analytical techniques established for fin surface temperature.
- Fin performance and multivariable optimization study for an actual design.
- Nonuniform convection coefficients on wet surface declined design performance well.

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ABSTRACT

An analytical model based on the Differential Transform Method and Adomian Decomposition Method is established for predicting the performance parameters and optimum design parameters of a wet T-shaped fin by considering temperature dependent thermal conductivity of the fin material and convective heat transfer coefficient. The mass transfer process is calculated by adopting a cubic variation of humidity ratio of saturated air with the corresponding fin surface temperature. From the optimization analysis, two types of optimized results are presented. First, the Biot number is taken as a constant and the optimum values of all the geometric parameters of a T-shaped fin (length ratio, thickness ratio and thickness to length ratio of stem) are determined. Next the optimized results are presented by taking the heat transfer rate and fin volume as functions of Biot number and only one of the three geometric parameters, the other two being constants. Also, the temperature distribution, fin performances and optimum design parameters obtained from the present analysis are compared with that from the published analysis and significant differences are noticed due to the superiority of the present study.

1. Introduction

In the recent years, the increased concern regarding material and energy savings has motivated the engineers and researchers to carry out extensive studies in the field of heat transfer enhancement techniques. One such widely employed technique is, attaching the base surface with extended surfaces which are called fins. The idea behind by using fins for improving the heat transfer rate is to increase the surface area, thereby allowing more heat to flow through them. Fins are extensively used in large number of equipment where the air is cooled when it is being passed over the fin surface, maintaining at a temperature lower than the ambient temperature. Examples of such kind can be easily seen in a number of industrial applications such as air-conditioning, refrigeration, and chemical industries. Again, in these applications, heat transfer will be accompanied by mass transfer if the fin surface is at a temperature lower than the dew point temperature of the air being cooled and in such cases the fin surface is covered with a thin film of water and fins under this condition are termed as wet fins. The fin surface may be fully wet or partially wet depending upon the psychometric condition of the incoming air and the fin surface temperature. The first case results when the temperature on the entire fin surface is lower than the dew point temperature of the air being cooled whereas partially wet fin results if the dew point temperature lies between fin tip and fin base temperature.

Extensive research works to analyze the performance of different types of fins under dehumidifying conditions have already been reported. The first initiative to study wet fins analytically

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Nomenclature

Bi	Biot number
$C_{p,a}$	specific heat of the ambient air at constant pressure (J kg^{-1} K^{-1})
G	gradient of dimensionless temperature at the junction
h	local convective heat transfer coefficient over the surface of the fin (W m^{-2} K^{-1})
h_B	convective heat transfer coefficient at the fin base (W m^{-2} K^{-1})
h_{fg}	latent heat of condensation of moisture (J kg^{-1})
h_m	mass transfer coefficient (kg m^{-2} s^{-1})
k	local thermal conductivity of the fin material (W m^{-1} K^{-1})
k_B	thermal conductivity of the fin material at the base temperature (W m^{-1} K^{-1})
l_F	half of the flange length (m)
l_S	stem length (m)
Le	Lewis number
m	index for variable heat transfer coefficient
q_A	actual heat transfer rate through the fin per unit width (W m^{-1})
Q_A	dimensionless actual heat transfer rate through the fin per unit width
q_I	ideal heat transfer rate through the fin per unit width (W m^{-1})
Q_I	dimensionless ideal heat transfer rate per unit width
q_{WF}	heat transfer rate through the fin per unit width in unfinned condition (W m^{-1})
Q_{WF}	dimensionless heat transfer rate per unit width in unfinned condition
RH	relative humidity of surrounding air
t_S	stem thickness (m)
t_F	flange thickness (m)
T	local fin surface temperature (°C)
T_A	ambient temperature (°C)
T_B	fin base temperature (°C)
U	dimensionless fin volume per unit width
V	fin volume per unit width (m^2)
w	fin width (m)
x	axial length measured from flange tip as shown in Fig. 1 (m)
X	dimensionless distance, x/l_F
y	axial length measured from the junction along the stem as shown in Fig. 1 (m)
Y	dimensionless length, y/l_S

Greek symbols

ω	local humidity ratio of the air on the fin surface (kg of water vapor per kg of dry air)
ω_A	humidity ratio of surrounding air (kg of water vapor per kg of dry air)
β	co-efficient of variable thermal conductivity (K^{-1})
ξ	dehumidification parameter, $h_{fg}/(C_{p,a}Le^{2/3})$ °C
ψ	length ratio, l_F/l_S
ψ^*	thickness ratio, t_F/t_S
τ	thickness to length ratio of stem, t_S/l_S
τ_F	thickness to length ratio of flange, t_F/l_F
η	fin efficiency
ε	fin effectiveness
ϕ_F	dimensionless temperature for the flange part of the fin, $(T_A - T_F)/(T_A - T_B)$
ϕ_S	dimensionless temperature for the stem part of the fin, $(T_A - T_S)/(T_A - T_B)$

Subscripts

A	ambient condition
B	fin base
F	flange
S	stem

was carried out by Threlkeld [1], where the overall fin efficiency of longitudinal fins of rectangular profile was determined by considering the air temperature to be linearly related to the corresponding saturated air enthalpy. Considering the same design condition, McQuiston [2] estimated the wet fin performance parameters, by considering the fact that the difference in humidity ratio was linearly related to the corresponding temperature difference. In the subsequent years, McQuiston's model for determining wet fin efficiency was modified by Xu et al. [3] by including the enthalpy change of the moving condensate film in the governing equation. Elmahdy and Biggs [4] determined numerically the temperature distribution over the fin surface and the overall fin efficiency of wet circular and longitudinal fins of uniform thickness by using a linear relation between the humidity ratio of saturated air and the corresponding fin surface temperature. Later on, considering the same linear relation, various researchers [5–12] had analyzed fins of different geometries with different profiles under simultaneous heat and mass transfer.

But this assumption of linear relation between humidity ratio of saturated air and the corresponding fin surface temperature is an approximation and can be used only for small difference of temperature between the fin base and fin tip. In actual case, when moisture condenses on the fin surface, the incoming air adjacent to the fin surface becomes saturated and hence the variation of humidity ratio of air on the fin surface with the corresponding saturation temperature follows the saturation curve on the psychometric chart. The saturation curve on the psychometric chart is a curvilinear in nature. Liang et al. [13] used a cubic relationship between the humidity ratio of saturated air and the corresponding fin surface temperature and presented a one-dimensional analytical model, a one-dimensional numerical model and a two-dimensional numerical model for determining the wet fin efficiency of a plate fin tube heat exchanger. Later on, numerous analyses [14–21] were carried out by considering this cubic variation of humidity ratio of saturated air with the corresponding fin surface temperature.

In every fin analysis problem, the determination of the optimum fin geometry is of paramount importance for implementing the fin in practical situations. Kundu and Das [5] analyzed the temperature distribution, performance parameters and optimum design parameters of longitudinal fins, annular fins and spines, having both trapezoidal and triangular profiles under fully wet condition. A generalized optimization method based on the Lagrange multiplier technique was demonstrated and either the heat transfer rate or the fin volume may be taken as a constraint depending upon the design requirements. Closed form analytical solution for the fin efficiency of a fully wet annular fin and for longitudinal fins of different profiles (rectangular, triangular, convex parabolic, concave parabolic) was presented by Sharqawy and Zubair [6,7]. Based on the analytical solution, certain regression equations were developed and from these equations, the optimum dimensions of the fin could easily be calculated when the fin volume or the heat transfer rate was specified. Analyses to determine the performance and optimum design of wet annular fins of