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Inverse trishear modeling of bedding dip data using Markov chain Monte Carlo methods

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ABSTRACT

We present a method for fitting trishear models to surface profile data, by restoring bedding dip data and inverting for model parameters using a Markov chain Monte Carlo method. Trishear is a widely-used kinematic model for fault-propagation folds. It lacks an analytic solution, but a variety of data inversion techniques can be used to fit trishear models to data. Where the geometry of an entire folded bed is known, models can be tested by restoring the bed to its pre-folding orientation. When data include bedding attitudes, however, previous approaches have relied on computationally-intensive forward modeling. This paper presents an equation for the rate of change of dip in the trishear zone, which can be used to restore dips directly to their pre-folding values. The resulting error can be used to calculate a probability for each model, which allows solution by Markov chain Monte Carlo methods and inversion of datasets that combine dips and contact locations. These methods are tested using synthetic and real datasets. Results are used to approximate multimodal probability density functions and to estimate uncertainty in model parameters. The relative value of dips and contacts in constraining parameters and the effects of uncertainty in the data are investigated.

1. Introduction

Trishear is a kinematic model for fault-propagation folds, in which a triangular zone of distributed deformation occurs ahead of the fault tip. It is capable of reproducing features such as non-uniform forelimb dips and footwall synclines (Erslev, 1991) — features that are not explained by kink band models (Suppe, 1985; Suppe and Medwedeff, 1990) but are frequently observed in the field. Deformation within the trishear zone can be described in terms of a velocity field (Hardy and Ford, 1997; Zehnder and Allmendinger, 2000) in which the velocity of a particle at any given point can be calculated from its position relative to the fault tip. By integrating this velocity through the slip of the fault as the fault tip propagates, the geometry of a trishear fold can be modeled from the initial stratigraphy and the trishear model parameters: tip position, fault dip, fault slip, trishear zone apical angle, propagation to slip ratio, and the concentration factor, which if greater than one concentrates deformation toward the center of the trishear zone. No analytic solution is known, but the integration can be performed numerically.

Balanced cross sections, although fundamental to structural geology, are often non-unique, and multiple interpretations may be possible. This is especially likely when subsurface data are absent. Interpretations based on trishear kinematics are by no means exempt from these concerns, and the six or seven (if the concentration factor is included) free parameters in the model make it likely that there will be a range of reasonable interpretations. Interpretations, such as shortening estimates based on a balanced cross section, may have substantial uncertainty (Judge and Allmendinger, 2011) and will be more useful if this uncertainty can be quantified. Trishear folds, which must be modeled numerically, are amenable to methods that require testing a large number of potential cross-sections. Such techniques have been used to find a best-fit trishear model for given data beginning with the work of Allmendinger (1998), who used a grid search over the parameter space. The grid search approach has subsequently been employed by other authors (e.g. Allmendinger and Shaw, 2000; Allmendinger et al., 2004; Cardozo, 2005; Lin et al., 2007), principally with the aim of identifying a best-fit model.

Quantifying uncertainty in trishear parameters presents a greater challenge than finding a best-fit model alone, but it is essential to a full understanding of a structure. Although not

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commonly done, formal uncertainty can be estimated from grid search results, as we demonstrate below. Other methods to quantify uncertainty in trishear model parameters have been proposed by Cardozo and Aanonsen (2009) and Regalla et al. (2010). Cardozo and Aanonsen (2009) use the randomized maximum likelihood (RML) method, in which the best fit is found by optimization for many different realizations of the data and the results are plotted as histograms for each trishear parameter. Regalla et al. (2010) use a Monte Carlo simulation and plot histograms of all models for which the objective function is below a certain threshold. Cardozo et al. (2011) have also shown that simulated annealing can be used to estimate the range of possible models. In this study, we propose the use of Markov chain Monte Carlo methods. Such methods, while not previously applied to trishear, are commonly used for data inversion and are capable of both finding a best fit and estimating uncertainty (Tarantola, 2005).

Trishear kinematics is fully reversible and so can be used to test possible cross-sections by restoration (Allmendinger, 1998). A bed trace, imaged in seismic data or exposed in outcrop along the length of the fold, can be restored in this manner, and the model results can be compared to a predicted geometry using an objective function (such as distance from a point to a best fit straight line) in order to evaluate the goodness of fit. Mapped contacts along a surface profile can also be restored and matched across the fold or to a known original depth, but dip measurements pose more of a problem. The approach used in previous work to match dip measurements to trishear models (Cardozo, 2005; Regalla et al., 2010) has been to forward model beds, interpolate dips between modeled points, and attempt to match the observed dips.

In summary, work by various authors has demonstrated the value of the inverse method, using a variety of inversion algorithms, in fitting trishear models to data. More limited, but significant, work has shown that the technique can be applied to datasets consisting of dips or of mixed dip and point data and has proposed some methods by which error in trishear parameters can be estimated. We build on this body of knowledge by developing a method for the direct restoration of dip data, testing Markov Chain Monte Carlo techniques for trishear data inversion and error estimation, and investigating the relative value of contact positions and bedding dips in constraining a trishear model.

2. Methods

2.1. Velocity equations

The trishear velocity field proposed by Zehnder and Allmendinger (2000), while not the only possible velocity field, is among the simplest and most widely used, often in its linear form ($s = 1$ in Eq. (1)) (Hardy and Allmendinger, 2011). This formulation is supported by the finite element modeling of Cardozo et al. (2003) for an elastoplastic material, which produces a similar fold shape. This velocity field is defined with reference to a coordinate system (x,y), for which the origin is at the fault tip and moves with it as the fault propagates. A second coordinate system (ζ,η) has its origin fixed at the initial fault tip position. The trishear zone is a triangular region defined by the angle (φ) between its boundaries and the x-axis. The tangent of φ is denoted m. In some cases, the trishear zone may be asymmetric, requiring two φ values (Zehnder and Allmendinger, 2000), but we will focus on the symmetric case. The hanging wall velocity outside the trishear zone is labeled v_0. Fig. 1 shows the geometry of the trishear zone with the coordinate axes and important variables indicated. The trishear velocity field of Zehnder and Allmendinger (2000) is:

$$v_x = \frac{v_0}{2}\left[\mathrm{sgn}(y)\left(\frac{|y|}{mx}\right)^{\frac{1}{s}} + 1\right]$$
$$v_y = \frac{v_0 m}{2(1+s)}\left[\left(\frac{|y|}{mx}\right)^{\frac{(1+s)}{s}} - 1\right] \quad (1)$$
$$-xm \le y \le xm, s \ge 1$$

where sgn(y) indicates the sign of y, and s is a concentration factor. Increasing s concentrates deformation toward the center of the trishear zone. When $s = 1$, the field is termed “linear” (Zehnder and Allmendinger, 2000) or “homogeneous trishear” (Erslev, 1991). This velocity field properly describes the velocities in the (ζ,η) coordinate system. In the (x,y) coordinate system there is an additional component of relative motion in the x direction, equal to $-v_0(P/S)$, due to fault propagation. P/S is the ratio of fault propagation to fault slip. In summary, there are seven parameters that determine the form of a trishear fold: the two coordinates of the fault tip (x_t, y_t), the fault ramp angle (fault dip), the total slip on the fault, P/S, ϕ, and s.

This velocity field can be used directly to restore a folded bed by incrementally moving points along the bed. For dip data, the forward modeling approach (Cardozo, 2005; Regalla et al., 2010), requires moving a large number of points for each dip, which is time-consuming. If dips are instead restored directly, via an equation for the rate of change of dip, dip data can be inverted in the same manner as point data. This approach is much more efficient, allows a restoration approach to be used with datasets that contain both dip and point (bed or contact) data, and also reduces the errors inherent in interpolating between points.

The velocity field of Eq. (1), or any trishear velocity field, can be used to define a strain rate tensor in two-dimensions:

$$\dot{\mathbf{e}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix} \quad (2)$$

If a dip in cross-section is defined by an angle that it makes with the x-axis in the trishear coordinate system, θ, then the strain rate tensor can be transformed into a coordinate system aligned with the dip:

$$\dot{\mathbf{e}}' = \mathbf{a}^T\dot{\mathbf{e}}\mathbf{a} \quad (3)$$

where $\dot{\mathbf{e}}'$ is the strain rate tensor in a Cartesian coordinate system with its x′-axis parallel to the dip, and $\mathbf{a}$ is the rotation matrix:

$$\mathbf{a} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (4)$$

The off diagonal term $\dot{\mathbf{e}}'_{21}$ of the rotated strain rate tensor will then be the rate of rotation of a line parallel to the x′-axis (Allmendinger et al., 2012) and will therefore be equal to the rate of change of dip.

$$\dot{\theta} = \dot{\mathbf{e}}'_{21} = -\frac{\partial v_x}{\partial x}\cos\theta\sin\theta + \frac{\partial v_y}{\partial x}\cos^2\theta - \frac{\partial v_x}{\partial v_y}\sin^2\theta + \frac{\partial v_y}{\partial y}\cos\theta\sin\theta \quad (5)$$

For the trishear velocity field of Eq. (1), this simplifies to

$$\dot{\theta} = -\frac{v_0}{2sx}\left[\sqrt{m}\left(\frac{|y|}{mx}\right)^{\frac{1+s}{2s}}\cos\theta - \mathrm{sgn}(y)\sqrt{\frac{1}{m}}\left(\frac{|y|}{mx}\right)^{\frac{1-s}{2s}}\sin\theta\right]^2 \quad (6)$$

Note that Eq. (6) uses the convention that θ is positive